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IMPROVEMENT IN ESTIMATING FINITE POPULATION MEAN USING AUXILIARY VARIABLE IN TWO-PHASE SAMPLING

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Abstract: The auxiliary information is used frequently by the survey sampling researchers at the estimation stage in order to develop efficient estimators from long decades. Besides the classical ratio, product and regression methods; exponential estimators are useful for estimating the finite population mean when there is low degree of correlation between study variable and auxiliary variable. In this regard, Singh and Vishwakarma [8] suggested a ratio type exponential estimator in two-phase sampling. Again, Searls [17] used coefficient of variation for improving mean per unit estimator. Srivastava [18] developed an estimator of population mean using estimated coefficient of variation of y from the sample value when population coefficient variation is not known. Further, Upadhyaya and Srivastava [19,20] have suggested an improved estimator of population mean $\bar{Y}$ in symmetrical population considering the estimated coefficient of variation of y. Motivated by these, the present paper proposes some improved ratio type exponential estimators of finite population mean under simple random sampling without replacement in two-phase sampling using auxiliary variable. The proposed estimators have been compared with the mean per unit estimator, the conventional two-phase ratio estimator and the two-phase ratio type exponential estimator suggested by Singh and Vishwakarma [8] both theoretically and empirically.

Keywords: *Two-phase sampling, Ratio type exponential estimators, Auxiliary variable, Bias, Efficiency.*

I INTRODUCTION

In survey sampling, one can use auxiliary information to improve the efficiency of the estimators. In sampling literature, the most commonly used estimators are ratio estimator (Laplace[1], Cochran[2]), regression estimator (Watson [3]) and product estimator (Robson [4], Murthy[5]). The ratio and product estimators are suggested when there exists a positive and a negative correlation between study variable and auxiliary variable respectively. The ratio, product and regression estimators are recommended when the linear relationship between study variable and auxiliary variable is very strong whereas, exponential estimators are preferred when the linear relationship is not very strong. Some contributions towards development of exponential estimators are due to Bahl and Tuteja [6], Kadilar and Chingi [7], Singh and Viswakarma [8], Singh et.al. [9], Singh et.al. [10], Sanaulla et.al. [11], Singh and Choudhary [12], Khan [13] and Tailor et al. [14] and many others.

Consider a finite population $U = \{1,2,3,\ldots\ldots,N\}$.Let y and x be the study variable and auxiliary variable respectively assuming the value of y_i and x_i on the i^{th} unit $i = \{1,2,3,\ldots\ldots,N\}$.Now consider y be the of study variable and x be the auxiliary variable. Further we assume that y and x are positively correlated. When the population mean $\bar{X}$ of the auxiliary variable x is not known in advance, then in order to take the advantage of the estimators involving $\bar{X}$, we use double sampling scheme. In a two-phase sampling scheme, the samples are selected at two phases as follows:

i. A large preliminary sample $s'(s' \subset U)$of size $n' < N$, called as the first phase sample, is selected from the population of size N to study the auxiliary variable x with a purpose to estimate the unknown population mean $\bar{X}$ of x by $\bar{x}'$.

ii. A second phase sample 's' of (relatively smaller) size $n\ (< n')$is selected in order to study both the study variable y as well as the auxiliary variable x.

Following Bose [15], we have considered that the second-phase sample is directly selected from the population of size N and is independent from the first-phase sample. At both the phases, the samples are selected by using SRSWOR scheme. Let $\bar{x},\bar{y}$ be the sample means of x and y basing upon the second phase sample and $\bar{x}'$ is the sample mean of x basing on the first phase sample. So,

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \qquad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \text{ and } \bar{x}' = \frac{1}{n'}\sum_{i=1}^{n'} x_i$$

In no information case, we use the mean per unit estimator $(\bar{y})$ to estimate $\bar{Y}$. The usual two-phase ratio estimator $t_{TR}$of $\bar{Y}$is given by

$$t_{TR} = \frac{\bar{y}}{\bar{x}}\bar{x}' \qquad (1)$$

and two-phase ratio type exponential estimator suggested by Singh and Vishwakarma [8] is given by

$$t_{TER1} = \bar{y}\ \mathrm{Exp}\left[\frac{\bar{x}' - \bar{x}}{\bar{x}' + \bar{x}}\right]. \qquad (2)$$

In two-phase sampling scheme if the second-phase sample is independent of the first- phase sample, then $\mathrm{Cov}(\bar{y},\bar{x}') = \mathrm{Cov}(\bar{x},\bar{x}') = 0$.

The bias and mean square error (MSE) of these two estimators$t_{TR}$and t_{TER1}upto $o(n^{-1})$are respectively given by

$$B(t_{TR}) = \bar{Y}\left[\theta_1\left(C_{20} - C_{11}\right)\right], \qquad (3)$$

$$\mathrm{MSE}\ (t_{TR}) = \bar{Y}^2\left[\theta_1(C_{20} + C_{02}) + \theta_1' C_{20} - 2\theta_1 C_{11}\right], \qquad (4)$$

$$B(t_{TER1}) = \bar{Y}\left[\theta_1\left(\frac{3}{8}C_{20} - \frac{1}{2}C_{11}\right) - \frac{1}{8}\theta_1' C_{20}\right], \qquad (5)$$

$$\mathrm{MSE}\ (t_{TER1}) = \bar{Y}^2\left[\theta_1\left(C_{02} + \frac{1}{4}C_{20} - C_{11}\right) + \frac{1}{4}\theta_1' C_{20}\right], \qquad (6)$$

where, $C_{rs} = \frac{\mu_{rs}(x,y)}{\bar{X}^r\bar{Y}^s}$; $\theta_1 = \left(\frac{1}{n} - \frac{1}{N}\right)$; $\theta_1' = \left(\frac{1}{n'} - \frac{1}{N}\right)$

and $\mu_{rs}(x,y)$ being the $(r,s)^{th}$ bivariate moment of x and y.

In this paper following the exponential estimator of Bahl and Tuteja [6];double sampling exponential estimator of Singh and Vishwakarma [8],using coefficient of variation by Searls [16], using estimated coefficient of variation by Srivastava [17] and Upadhyaya and Srivastava [18,19] ,we have proposed three ratio type exponential estimators in two-phase sampling. These estimators are compared with the mean per unit estimator $(\bar{y})$, conventional two-phase ratio estimator (t_{TR}) and two-phase ratio-type exponential estimator (t_{TER1})both theoretically and empirically.

II. PROPOSED ESTIMATORS

In two-phase sampling scheme with independent samples, we have proposed following three modified exponential ratio type estimator to estimate population mean $\bar{Y}$

$$t_{TER2} = \frac{\bar{y}}{1 + \theta_1 C_{02}}\ \mathrm{Exp}\left[\frac{\bar{x}' - \bar{x}}{\bar{x}' + \bar{x}}\right]$$

where, C_{02},the square of population coefficient of variation of y and we assume that it is known .

$$t_{TER3} = \frac{\bar{y}}{1 + \theta_1 \hat{C}_{02}}\ \mathrm{Exp}\left[\frac{\bar{x}' - \bar{x}}{\bar{x}' + \bar{x}}\right]$$

where,$\hat{C}_{02}$, the square of sample coefficient of variation of y.

$$t_{TER4} = \bar{y}(1 + \theta_1 \hat{C}_{02})\ \mathrm{Exp}\left[\frac{\bar{x}' - \bar{x}}{\bar{x}' + \bar{x}}\right],$$

where, $S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})^2$and $s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$

III. BIAS AND MSE OF PROPOSED ESTIMATORS

In order to find the bias and MSE of these proposed estimators, denote

$$\delta_{\bar{y}} = \frac{\bar{y}-\bar{Y}}{\bar{Y}}\ ,\ \ \delta_{\bar{x}} = \frac{\bar{x}-\bar{X}}{\bar{X}}\ \text{ and } \delta_{\bar{x}'} = \frac{\bar{x}'-\bar{X}}{\bar{X}},$$

where $E(\delta_{\bar{y}}) = E(\delta_{\bar{x}}) = E(\delta_{\bar{x}'}) = 0$ and

$E(\delta_{\bar{y}}^{\,2}) = \theta_1 C_{02}$, $E(\delta_{\bar{x}}^{\,2}) = \theta_1 C_{20}$,

$E(\delta_{\bar{x}'}^2) = \theta_1' C_{20}$, $E(\delta_{\bar{y}}\delta_{\bar{x}}) = \theta_1 C_{11}$,

$E(\delta_{\bar{x}'}\delta_{\bar{y}}) = \theta_1' C_{11}$, $E(\delta_{\bar{x}'}\delta_{\bar{x}}) = \theta_1' C_{20}$

Assuming the validity of Taylor's series expansion of t_{TER1}, t_{TER2}, t_{TER3} and t_{TER4} we get

$$t_{TER1} = \bar{Y}\left[1+\delta_{\bar{y}}-\frac{1}{2}\delta_{\bar{x}}+\frac{1}{2}\delta_{\bar{x}'}-\frac{1}{2}\delta_{\bar{y}}\delta_{\bar{x}}+\frac{3}{8}\delta_{\bar{x}}^2-\frac{1}{8}\delta_{\bar{x}'}^2+\frac{1}{2}\delta_{\bar{y}}\delta_{\bar{x}'}-\frac{1}{4}\delta_{\bar{x}}\delta_{\bar{x}'}+o(n^{-2})\right]$$

$$t_{TER2} = \bar{Y}\Big[1+\delta_{\bar{y}}-\frac{1}{2}\delta_{\bar{x}}-\frac{1}{2}\delta_{\bar{y}}\delta_{\bar{x}}+\frac{3}{8}\delta_{\bar{x}}^2+\frac{1}{2}\delta_{\bar{x}'}-\frac{1}{8}\delta_{\bar{x}'}^2+\frac{1}{2}\delta_{\bar{y}}\delta_{\bar{x}'}-\frac{1}{4}\delta_{\bar{x}}\delta_{\bar{x}'}-\theta_1C_{02}-\theta_1C_{02}(\delta_{\bar{y}} -\frac{1}{2}\delta_{\bar{x}}+\frac{1}{2}\delta_{\bar{x}'})+o(n^{-2})\Big]$$

$$t_{TER3} = \bar{Y}\Big[1+\delta_{\bar{y}}-\frac{1}{2}\delta_{\bar{x}}-\frac{1}{2}\delta_{\bar{y}}\delta_{\bar{x}}+\frac{3}{8}\delta_{\bar{x}}^2+\frac{1}{2}\delta_{\bar{x}'}-\frac{1}{8}\delta_{\bar{x}'}^2+\frac{1}{2}\delta_{\bar{y}}\delta_{\bar{x}'}-\frac{1}{4}\delta_{\bar{x}}\delta_{\bar{x}'}-\theta_1C_{02}-\theta_1C_{02}(\delta_{\bar{y}} -\frac{1}{2}\delta_{\bar{x}}+\frac{1}{2}\delta_{\bar{x}'})+o(n^{-2})\Big]$$

$$t_{TER4} = \bar{Y}\Big[1+\delta_{\bar{y}}-\frac{1}{2}\delta_{\bar{x}}-\frac{1}{2}\delta_{\bar{y}}\delta_{\bar{x}}+\frac{3}{8}\delta_{\bar{x}}^2+\frac{1}{2}\delta_{\bar{x}'}-\frac{1}{8}\delta_{\bar{x}'}^2+\frac{1}{2}\delta_{\bar{y}}\delta_{\bar{x}'}-\frac{1}{4}\delta_{\bar{x}}\delta_{\bar{x}'}+\theta_1C_{02}+\theta_1C_{02}(\delta_{\bar{y}} -\frac{1}{2}\delta_{\bar{x}}+\frac{1}{2}\delta_{\bar{x}'})+o(n^{-2})\Big]$$

Considering the expected value to $o(n^{-1})$, the bias of the different estimators are

$$B(t_{TER2}) = E(t_{TER2})-\bar{Y} = \bar{Y}\left[\theta_1\left(\frac{3}{8}C_{20}-\frac{1}{2}C_{11}-C_{02}\right)-\frac{1}{8}\theta_1'C_{20}\right] \quad (10)$$

$$B(t_{TER3}) = E(t_{TER3})-\bar{Y} = \bar{Y}\left[\theta_1\left(\frac{3}{8}C_{20}-\frac{1}{2}C_{11}-C_{02}\right)-\frac{1}{8}\theta_1'C_{20}\right] \quad (11)$$

$$B(t_{TER4}) = E(t_{TER4})-\bar{Y} = \bar{Y}\left[\theta_1\left(\frac{3}{8}C_{20}-\frac{1}{2}C_{11}+C_{02}\right)-\frac{1}{8}\theta_1'C_{20}\right] \quad (12)$$

However, when the sample size is very large the contribution of bias of proposed estimators may be negligible for all practical purposes. The mean square error of estimator t_{TER1} up to $o(n^{-1})$are given by

$$MSE(t_{TER1}) = E(t_{TER1}-\bar{Y})^2 = \bar{Y}^2E\left[\delta_{\bar{y}}-\frac{1}{2}\delta_{\bar{x}}+\frac{1}{2}\delta_{\bar{x}'}\right]^2$$

$$= \bar{Y}^2E\left[\delta_{\bar{y}}^2+\frac{1}{4}\delta_{\bar{x}'}^2+\frac{1}{4}\delta_{\bar{x}}^2+\delta_{\bar{y}}\delta_{\bar{x}'}-\delta_{\bar{y}}\delta_{\bar{x}}-\frac{1}{2}\delta_{\bar{x}}\delta_{\bar{x}'}\right]$$

$$= \bar{Y}^2\left[\theta_1\left(C_{02}+\frac{1}{4}C_{20}-C_{11}\right)+\frac{1}{4}\theta_1'C_{20}\right]$$

It can easily be seen that that the mean square error of estimators t_{TER2}, t_{TER3} and t_{TER4} up to $o(n^{-1})$ are equal to mean square error of t_{TER1} given in (6). So, in order to study the efficiency of the estimators t_{TER1}, t_{TER2}, t_{TER3} and t_{TER4} , the mean square error of these estimators have been derived upto $o(n^{-2})$.The mean square errors (MSEs) of different estimators to $o(n^{-2})$are derived as

$$MSE(t_{TER1}) = \bar{Y}^2[\{\theta_1\left(\frac{1}{4}C_{20}+C_{02}-C_{11}\right)+\frac{1}{4}\theta_1'C_{02}\}$$
$$+\left\{\left(\theta_2-\frac{3\theta_1}{N}\right)\left(\frac{5}{4}C_{21}-\frac{3}{8}C_{30}-C_{12}\right)-\frac{1}{8}\left(\theta_2'-\frac{3}{N}\theta_1'\right)C_{30}\right\}$$
$$+\{\theta_1^2\left(\frac{79}{64}C_{20}^2+C_{20}C_{02}+2C_{11}^2-\frac{31}{8}C_{11}C_{20}\right)+\frac{7}{64}\theta_1'^{\,2}C_{20}^2\}], \quad (13)$$

where, $\theta_2 = (\frac{1}{n^2}-\frac{1}{N^2})$,$\theta_2' = (\frac{1}{n'^2}-\frac{1}{N^2})$.

$$MSE(t_{TER2}) = MSE(t_{TER1})-\bar{Y}^2[\frac{1}{4}\theta_1\theta_1'C_{20}C_{02}-\theta_1^2\left(3C_{11}C_{02}-\frac{5}{4}C_{02}C_{20}-C_{02}^2\right)] \quad (14)$$

$$MSE(t_{TER3}) = MSE(t_{TER1})-\bar{Y}^2\left[\frac{1}{4}\theta_1\theta_1'C_{20}C_{02}-\theta_1^2\left(C_{11}C_{02}-\frac{5}{4}C_{02}C_{20}+3C_{02}^2-2C_{03}+C_{12}\right)\right] (15)$$

$$MSE(t_{TER4}) = MSE(t_{TER1})-\bar{Y}^2\left[\theta_1^2\left(C_{12}-\frac{5}{4}C_{02}C_{20}+C_{11}C_{02}+C_{02}^2-2C_{03}\right)-\frac{1}{4}\theta_1\theta_1'C_{20}C_{02}\right] (16)$$

IV COMPARISON OF EFFICIENCY

It is seen that the mean square errors of t_{TER1}, t_{TER2}, t_{TER3} and t_{TER4} up to $o(n^{-1})$are same. So,the mean square error of the estimators was considered up to $o(n^{-2})$ for comparison of efficiency.From (13), (14), (15) and (16) we get the following.

I. The estimator t_{TER2} is more efficient than t_{TER1} if

$$C_{11} < \frac{1}{12\theta_1}[\theta_1' C_{20} + 5\theta_1 C_{20} + 4\theta_1 C_{02}] \quad (17)$$

Assuming the symmetrical bivariate distribution of (x ,y) the inequality (15) reduces to

$$\rho < \frac{1}{12\theta_1 Z}[\theta_1' Z^2 + 5\theta_1 Z^2 + 4\theta_1],$$

where, $Z = \left(\frac{C_{20}}{C_{02}}\right)^{\frac{1}{2}}$ and ρ is the correlation coefficient between x and y for the population.

II. The estimator t_{TER3} is more efficient than t_{TER1} if

$$C_{11} < \frac{1}{4\theta_1 C_{02}}[\theta_1' C_{20} C_{02} + 5\theta_1 C_{20} C_{02} + 8\theta_1 C_{03} - 12\theta_1 C_{02}^2 - 4\theta_1 C_{12}] \quad (19)$$

Assuming symmetrical bivariate distribution of (x ,y) the inequality (17) reduces to

$$\rho < \frac{1}{4\theta_1 Z}[\theta_1' Z^2 + 5\theta_1 Z^2 - 12\theta_1]$$

III. The estimator t_{TER4} is more efficient than t_{TER1} if

$$C_{11} > \frac{1}{4\theta_1 C_{02}}[\theta_1' C_{20} C_{02} + 5\theta_1 C_{20} C_{02} + 8\theta_1 C_{03} - 4\theta_1 C_{02}^2 - 4\theta_1 C_{12}] \quad (21)$$

Assuming symmetrical bivariate distribution of (x ,y) the inequality (19) reduces to

$$\rho > \frac{1}{4\theta_1 Z}[\theta_1' Z^2 + 5\theta_1 Z^2 - 4\theta_1] \quad (22)$$

IV. The estimator t_{TER3} is more efficient than t_{TER2} if

$$C_{11} > \frac{1}{2C_{02}}[4C_{02}^2 + C_{12} - 2C_{03}] \quad (23)$$

Assuming symmetrical bivariate distribution of (x ,y) the inequality (21) reduces to

$$\rho > \frac{2}{Z} \quad (18) \quad (24)$$

V. The estimator t_{TER4} is more efficient than t_{TER2} if

$$C_{11} > \frac{1}{8\theta_1 C_{02}}[\theta_1' C_{20} C_{02} + 5\theta_1 C_{20} C_{02} + 4\theta_1 C_{03} - 2\theta_1 C_{12}] \quad (25)$$

Assuming symmetrical bivariate distribution of (x ,y) the inequality (23) reduces to

$$\rho > \frac{Z}{8\theta_1}(\theta_1' + 5\theta_1) \quad (26)$$

VI. The estimator t_{TER4} is more efficient than t_{TER3} if

$$C_{11} > \frac{1}{4\theta_1 C_{02}}[\theta_1' C_{20} C_{02} + 5\theta_1 C_{20} C_{02} + 8\theta_1 C_{03} - 8\theta_1 C_{02}^2 - 4\theta_1 C_{12}] \quad (27)$$

(20)

Assuming symmetrical bivariate distribution of (x ,y) the inequality (25) reduces to

$$\rho > \frac{1}{4\theta_1 Z}[\theta_1' Z^2 + 5\theta_1 Z^2 - 8\theta_1]$$

Empirical Study

To study the efficiency of different estimators we have considered eight natural populations from different text books. The comparison is based on percent relative efficiency

Pop^n No.	Ref.	X	Y
1	[20] p.541	Priceof Aluminum (in cents/pound)	Price of Gold (in dollars/ounce)
2	[20] p.541	Price of Copper (in cents/pound)	Price of Gold (in dollars/ounce)
3	[20] p.541	Cups of Coffee per day	Amount of Prepaid Card (in dollars)
4	[21] p.325	Number of Rooms	Number of Persons
5	[22] p.352	Operating Days per Month	Pounds of Steam Used per Month
6	[21] p.325	Operating Days per Month	Pounds of Fatty Acid Stored per Month
7	[21] p.325	Number of Startups	Pounds of Fatty Acid Stored per Month
8	[21] p.325	Weight Percent of Tricalcium Silicate	Weight Percent of Tricalcium Aluminate

of estimators with respect to mean per unit estimator. Table 1 gives the descriptions of population with Correlation Coefficient ρ and the Coefficient of Variation C_{20} and C_{02}. Table 2 gives the percent relative bias of the estimators. Table 3 gives the percent relative efficiency of different estimators $t_{TR} = \frac{\bar{y}}{\bar{x}}\bar{x}'$, t_{TER1}, t_{TER2}, t_{TER3} and t_{TER4} with respect to mean per unit estimator $t_0(=\bar{y})$.

Table 2. Population Parameters

Pop^n No.	Ref.	N	n'	n	ρ	C_{20}	C_{02}
1	[20] p.541	12	8	4	0.48	0.214	0.277
2	[20] p.541	12	8	4	0.42	0.258	0.277
3	[20] p.541	25	14	7	0.29	0.504	1.291
4	[21] p.325	10	7	4	0.65	0.135	0.152
5	[22] p.352	25	14	7	0.53	0.149	0.173
6	[21] p.325	25	14	7	0.69	0.149	0.15
7	[21] p.325	25	14	7	0.62	0.197	0.15
8	[21] p.325	13	8	4	0.23	0.323	0.788

Table 3. Percent Relative Bias of Estimators

	Estimators				
Pop. No.	t_{TR}	t_{TER1}	t_{TER2}	t_{TER3}	t_{TER4}
1	2.4318	0.2359	11.747	11.711	11.974
2	4.5036	1.1824	10.414	10.204	12.723
3	1.6256	0.1903	45.39	50.892	32.071
4	1.244	0.1556	6.9824	6.964	6.6481
5	1.4878	0.1176	6.0485	6.0691	6.2253
6	1.4879	0.0468	5.934	5.879	5.8697
7	3.3702	0.9606	4.4744	4.4401	6.4125
8	2.2543	0.3291	33.163	30.108	30.619

Table 4. Relative Efficiency(%) of Estimators w.rt $\bar{y}$

PNo.	t_{TR}	t_{TER1}	t_{TER2}	t_{TER3}	t_{TER4}
1	106.77	132.24	133.51	132.7	128.16
2	82.928	123.58	125.18	120.19	123.78
3	101.64	109.67	129.08	162.27	65.63
4	129.10	164.12	164.35	163.49	163.23
5	99.113	133.67	133.97	134.89	131.48
6	110.62	162.11	162.26	159.27	163.9
7	63.117	138.01	138.36	136.25	138.98
8	100.37	110.8	122.51	100.98	100.2

V CONCLUSIONS

From the Table 3 and Table 4 given above, we summarize the results as follows:

For populations 1, 2, 4, 5, 6, 7 and 8, the estimators,t_TER1, t_TER2, t_TER3 and t_(TER4)are more efficient than the mean per unit estimator $t0 = \bar{y}$.

For populations 1,2,4,5,6 and 7 the estimators,t_TER1, t_TER2, t_TER3 and t_(TER4)are more efficient than the classical two-phase ratio estimator t_TR .

For population 1, 2, 4 and 8 the estimator t_TER2 is most efficient.

For populations 3 and 5 the estimator t_TER3 is most efficient.

For population 6 and 7 the estimator t_TER4 is most efficient.

As the proposed estimator t_TER2 performs better than other estimators in most of the populations considered here, so it may be used as dominant estimator over other competitive estimators.

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