OPEN ACCESS INTERNATIONAL JOURNAL OF SCIENCE & ENGINEERING

COMPARISON OF FSDT AND SFSDT ON NANO RECTANGULAR FG PLATE VIBRATION BASED ON MODIFIED COUPLE STRESS THEORY UNDER MOVING LOAD

Hashemian. M, Amin. Y
Department of Mechanical Engineering, Khomeinishahr Branch, Islamic Azad University, Khomeinishahr / Isfahan, Iran
**Master of Mechanical Engineering, Khomeinishahr / Isfahan, Iran*
Yunesamin@gmail.com

Abstract: **In present paper, comparison of First Order Shear Deformation Theory (FSDT) and simple FSDT on Nano rectangular FG plate vibration analysis according to modified couple stress theory under moving load is developed. Every theory has identical pattern. First, displacement fields and associated strains are introduced. Equations of motion and boundary conditions are extracted from Hamilton's principle and for studying of small scale effect; modified couple stress theory is employed. Analytical solution for a simply supported Nano FG plate attained. Numerical examples are presented in order to verification of the research accuracy. The results show that power law index and length scale parameter have inverse effect on plate's deflections, but same trends in frequency, although there are slight differences between the theories at their amplitudes.**

Keywords *– Simple First Order Shear Deformation Theory, FSDT, Modified Couple Stress Theory, Hamilton's principle*

I INTRODUCTION

Functionally Graded Materials (FGMs) are a sort of composites which have continuous properties changing from one surface to another, thus they can be employed for a specific target. The prevalence of FGMs than typical composites is elimination of stress concentration which is emerging in laminated composites. FGMs are consisting of metal and ceramic, isotropic and non-homogenous. Utilization for specific stiffness and strength is because of smooth and continuous FGM's gradient. For the purpose of FGMs response prediction, accurate models are needed; there for shear deformation theories have been employed to take response as a result of acceptable response to FG plates. Owing to less complication and having valid results, FSDT has dragged many attentions. By increasing of engineering fields and attention to micro/nano structures, size dependent methods should take into consideration. Classical plate models are according to the classical continuum which could not calculate of size effect, because it has lack of length scale parameter, hence, size dependent models based on continuum length dependent deployed. Among size dependent theories, modified couple stress theory is more advantageous to the others, because it has a length scale parameter[1], [2], [3].There are many researches done based on theories which are introduced. Van vu et al. investigated simple FSDT based meshfree method for analyses of fg plates[4]. Jooybar et al. researched thermal effect on free vibration of fg truncated conical shell panel[5]. Reddy et al studied nonlinear finite element analysis of FG circular plates with modified couple stress theory[6]. Mirsalehi et al. studied the stability of thin FG micro-plates subjected to mechanical and thermal loading using modified couple stress and spline finite strip method[7]. Lei et al. studied A size dependent FG micro-plates model incorporating higher order shear and normal deformation effects based on a modified couple stress theory[8]. A simple shear deformation theory for nonlocal beams developed by Thai et.al[9]. Talha and Gupta investigated nonlinear flexural and vibration response of geometrically imperfect gradient plates using hyperbolic higher order shear and normal deformation theory[10]. Senjanovic et al., studied, on new first order shear deformation plate theories[11]. New first order shear deformation beam theory with in plane shear influence also investigated by Senjanovic et al[12]. Isogeometric locking free plate element: a simple first order shear deformation theory for fg plates is done by Yin[13].

Rohwer studied remark on a simple first order shear deformation theory for laminated composite plates[14]. An efficient and simple higher order shear and normal deformation theory for functionally graded material plates studied by Belabed et al[15]. Thai & Choi introduced a simple first order shear deformation theory for laminated composite plates[16]. A simple first order shear deformation theory for bending and free vibration analysis of functionally graded plates investigated by Thai and Choi[3]. Hence, the aims is comparing of vibration analysis of Nano FG rectangular plate based on FSDT and SFSDT under moving load. Either methods has identical pattern. First part solved based on FSDT and second's by simple FSDT. Displacement fields of each theory introduced as beginning one, then Equations of motion and boundary conditions are obtained from Hamilton's principle and modified couple stress theory. Analytical solution and numerical results for a simply supported plate indicated to evaluate of the research accuracy. Figure 1 shows the plate's specifications.

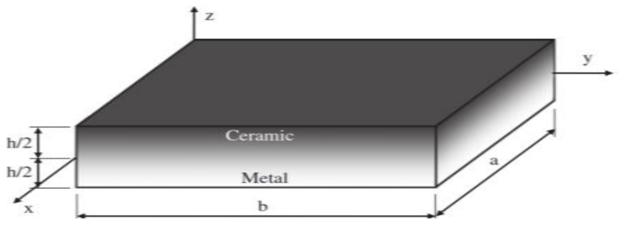

Figure 1: Geometry of FGM plate[17]

Section 1: FSDT relations

1. Displacement Fields

Displacement fields of FSDT can be written as [18]:

$$u_1(x,y,z,t)=u(x,y,t)+z\,\phi_x$$
$$u_2(x,y,z,t)=v(x,y,t)+z\,\phi_y \qquad (1)$$
$$u_3(x,y,z,t)=w(x,y,t)$$

In Eq. (1), u, v, w, ϕ_x , ϕ_y are the unknown displacement fields of the midplane of the plate.

are also defined as: $\phi_x=-\frac{\partial\theta}{\partial x}$ and $\phi_y=-\frac{\partial\theta}{\partial y}$

non zero strain equations based on FSDT are shown in Eq.(2)[19]:

$$\varepsilon_x=\frac{\partial u}{\partial x}+z\frac{\partial\phi_x}{\partial x}$$
$$\varepsilon_y=\frac{\partial v}{\partial y}+z\frac{\partial\phi_y}{\partial y}$$
$$\gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+z\left(\frac{\partial\phi_x}{\partial y}+\frac{\partial\phi_y}{\partial x}\right)$$
$$\gamma_{xz}=\phi_x+\frac{\partial w}{\partial x}$$
$$\gamma_{yz}=\phi_y+\frac{\partial w}{\partial y}$$
$$\varepsilon_z=0$$
. (2)

2. Curvature function

The curvature function according to rotation vectors are as Eq.(3)[3]:

$$\chi_{ij}=\frac{1}{2}\left(\frac{\partial\Theta_i}{\partial x_j}+\frac{\partial\Theta_j}{\partial x_i}\right)$$
$$i,j=1,2,3$$
. (3)

In Eq.(3), θ is rotation vector. Eq.(4) shows rotation vectors using displacement fields are as[20]:

$$\theta_1=\frac{1}{2}\left(\frac{\partial u_3}{\partial x_2}-\frac{\partial u_2}{\partial x_3}\right)$$
$$\theta_2=\frac{1}{2}\left(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1}\right) \qquad (4)$$
$$\theta_3=\frac{1}{2}\left(\frac{\partial u_2}{\partial x_1}-\frac{\partial u_1}{\partial x_2}\right)$$

Substituting Eq.(1) in Eq.(4), Eq.(5) exhibits rotation vectors dependent upon displacement fields.

$$\theta_x=\frac{1}{2}\left(\frac{\partial w}{\partial y}-\phi_y\right)$$
$$\theta_y=\frac{1}{2}\left(\phi_x-\frac{\partial w}{\partial x}\right)$$
$$\theta_z=\frac{1}{2}\left(\left(-\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+z\left(-\frac{\partial\phi_x}{\partial y}+\frac{\partial\phi_y}{\partial x}\right)\right)$$
(5)

Replacing of Eq.(5) in Eq.(3), Eq.(6) pointing out the curvature functions in terms of displacement fields:

$$\chi_{xx}=\frac{1}{2}\left(\frac{\partial^2 w}{\partial x\partial y}-\frac{\partial\phi_y}{\partial x}\right)$$
$$\chi_{yy}=\frac{1}{2}\left(\frac{\partial\phi_x}{\partial y}-\frac{\partial^2 w}{\partial x\partial y}\right)$$
$$\chi_{xy}=\frac{1}{4}\left(\frac{\partial^2 w}{\partial y^2}-\frac{\partial^2 w}{\partial x^2}+\frac{\partial\phi_x}{\partial x}-\frac{\partial\phi_y}{\partial y}\right)$$
$$\chi_{xz}=\frac{1}{4}\left((\frac{\partial^2 v}{\partial x^2}-\frac{\partial^2 u}{\partial x\partial y})+z(\frac{\partial^2\phi_y}{\partial x^2}-\frac{\partial^2\phi_x}{\partial x\partial y})\right)$$
$$\chi_{yz}=\frac{1}{4}\left((\frac{\partial^2 v}{\partial x\partial y}-\frac{\partial^2 u}{\partial y^2})+z(\frac{\partial^2\phi_y}{\partial x\partial y}-\frac{\partial^2\phi_x}{\partial y^2})\right)$$
$$\chi_{zz}=\frac{1}{2}\left(\frac{\partial\phi_y}{\partial x}-\frac{\partial\phi_x}{\partial y}\right)$$
(6)

3. Equations of Motion

Based on the Hamilton's principle, the equations of motions are as follows[20]:

$$\int_0^T (\delta U + \delta W - \delta K)dt = 0 \tag{7}$$

In which: δU : virtual strain energy: δW : virtual work done by external force; and δK : virtual kinetic energy. Virtual work defined as [21]:

$$\delta W = \int F\delta(x - x_0(t))\delta(y - y_0(t))\,dAdw = \int F\delta(x - x_0)\delta(y - y_0)\,dAdw \tag{8}$$

Also, virtual kinetic energy can be calculated as[19] :

$$\delta k = \int_A \int_{-h/2}^{h/2} \rho(z)\left(\dot{u}_1\delta\dot{u}_1 + \dot{u}_2\delta\dot{u}_2 + \dot{u}\delta\dot{u}_3\right)dAdz = \int_A \left[I_0\left(\dot{u}\delta\dot{u} + \dot{v}\delta\dot{v} + \dot{w}\delta\dot{w}\right) + I_1\left(\dot{u}\delta\dot{\varphi}_x + \dot{\varphi}_x\delta\dot{u} + \dot{v}\delta\dot{\varphi}_y + \dot{\varphi}_y\delta\dot{v}\right)\right] \left[+ I_2\left(\dot{\varphi}_x\delta\dot{\varphi}_x + \dot{\varphi}_y\delta\dot{\varphi}_y\right)\right]dA \tag{9}$$

Shear strain energy is take into consideration on the basis of modified couple stress theory as [22]:

$$U = \int_V (\sigma_{ij}\delta\varepsilon_{ij} + m_{ij}\delta\chi_{ij})\, dV \tag{10}$$

In Eq.(10): σ_{ij} are Cartesian components of the stress tensor, ε_{ij} are the strain components, m_{ij} are the components of deviatoric part of symmetric couple stress tensor and χ_{ij} are the components of the symmetric curvature tensor. By replacing of related relations into shear strains energy, Eq.(11) are attainable:

$$\begin{aligned}
\delta U = &\int \sigma_x\delta\varepsilon_x + \sigma_y\delta\varepsilon_y + \sigma_z\delta\varepsilon_z + \sigma_{xy}\delta\gamma_{xy} + \sigma_{xz}\delta\gamma_{xz} \\
&+ \sigma_{yz}\delta\gamma_{yz} + m_x\delta\chi_{xx} m_y\delta\chi_{yy} + m_z\delta\chi_{zz} + 2m_{xy}\delta\chi_{xy} \\
&+ 2m_{xz}\delta\chi_{xz} + 2m_{yz}\delta\chi_{yz} = \delta U = \int \sigma_{ij}\delta\varepsilon_{ij} + m_{ij}\delta\chi_{ij} \\
= &\int \sigma_x\delta\left(\frac{\partial u}{\partial x} + z\frac{\partial \phi_x}{\partial x}\right) + \sigma_y\delta\left(\frac{\partial v}{\partial y} + z\frac{\partial \phi_y}{\partial y}\right) + \\
&\sigma_{xy}\delta\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + z\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right)\right) \\
&+ \sigma_{xz}\delta\left(\frac{\partial w}{\partial x} + \phi_x\right) + \sigma_{yz}\delta\left(\phi_y + \frac{\partial w}{\partial y}\right) + \\
&\frac{1}{2}m_x\delta\left(\frac{\partial^2 w}{\partial y\partial x} - \frac{\partial \phi_y}{\partial x}\right) + \frac{1}{2}m_y\delta\left(\frac{\partial \phi_x}{\partial y} - \frac{\partial^2 w}{\partial x\partial y}\right) + \\
&\frac{1}{2}m_{xy}\delta\left(\frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w}{\partial x^2} + \frac{\partial \phi_x}{\partial x} - \frac{\partial \phi_y}{\partial y}\right) + \\
&+ \frac{1}{2}m_{xz}\delta\left(\left(\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 u}{\partial x\partial y}\right) + z\left(\frac{\partial^2 \phi_y}{\partial x^2} - \frac{\partial^2 \phi_x}{\partial x\partial y}\right)\right) + \\
&\frac{1}{2}m_z\left(\frac{\partial \varphi_y}{\partial x} - \frac{\partial \varphi_x}{\partial y}\right) + \\
&\frac{1}{2}m_{yz}\delta\left(\left(\frac{\partial^2 v}{\partial x\partial y} - \frac{\partial^2 u}{\partial y^2}\right) + z\left(\frac{\partial^2 \phi_y}{\partial x\partial y} - \frac{\partial^2 \phi_x}{\partial y^2}\right)\right)dAdz \\
\delta U = &\int N_x\delta\frac{\partial u}{\partial x} + M_x\delta\frac{\partial \phi_x}{\partial x} + N_y\delta\frac{\partial v}{\partial y} + M_y\delta\frac{\partial \phi_y}{\partial y} \\
&+ N_{xy}\delta\frac{\partial v}{\partial x} + Q_y\delta\frac{\partial w}{\partial y} + N_{xy}\delta\frac{\partial u}{\partial y} + \\
&M_{xy}\left(\frac{\delta\partial\phi_x}{\partial y} + \frac{\delta\partial\phi_y}{\partial x}\right) + Q_x\delta\frac{\partial w}{\partial x} + \\
&Q_y\delta\partial\phi_y + \frac{P_x}{2}\left(\frac{\partial w\delta w}{\partial x\partial y} - \delta\frac{\partial \phi_y}{\partial x}\right) \\
&+ \frac{P_z}{2}\left(\frac{\partial\delta\varphi_y}{\partial x} - \frac{\partial\delta\varphi_x}{\partial y}\right) + \frac{P_y}{2}\left(\delta\frac{\partial\phi_x}{\partial y} - \frac{\partial w\delta w}{\partial x\partial y}\right) + \\
&\frac{P_{xz}}{2}\left(\frac{\partial v\delta v}{\partial x^2} - \frac{\partial u\delta u}{\partial x\partial y}\right) + Q_x\delta\partial\phi_x + \\
&\frac{P_{xy}}{2}\left(\frac{\partial w\delta w}{\partial y^2} - \frac{\partial w\delta w}{\partial x^2} + \frac{\partial\delta\phi_x}{\partial x} - \frac{\partial\delta\phi_y}{\partial y}\right) \\
&+ \frac{R_{xz}}{2}\left(\frac{\partial\phi_y\delta\phi_y}{\partial x^2} - \frac{\partial\phi_x\delta\phi_x}{\partial x\partial y}\right) + \frac{R_{yz}}{2}\left(\frac{\partial\phi_y\delta\phi_y}{\partial x\partial y} - \frac{\partial\phi_x\delta\phi_x}{\partial y^2}\right) \\
&+ \frac{P_{yz}}{2}\left(\frac{\partial v\delta v}{\partial x\partial y} - \frac{\partial u\delta u}{\partial y^2}\right)dA
\end{aligned} \tag{11}$$

Regarding the FSDT theory, stress resultants, mass inertia and modified couple stress resultants of rectangular Nano FG plate are defined as follows. In order to simplify Eq. (11), by replacing of Eqs. (12-15) into it:

$$I_n = \int_{-h/2}^{h/2} \rho\left(z^n\right)dz \qquad n = 1,2,3 \tag{12}$$

$$N_i, M_i = \sigma_i \int_{-h/2}^{h/2} (1,z)dz, \quad i = x, y, xy \tag{13}$$

$$R_j = \int_{-h/2}^{h/2} zm_j dz,\; Q_j = k\int_{-h/2}^{h/2} \sigma_j dz \quad j = xz, yz \tag{14}$$

$$P_g = \int_{-h/2}^{h/2} m_g dz \quad g = x, y, xy, yz, xz \tag{15}$$

4. Governing Equations of Motion

Equations of motions are obtained by integration and collecting coefficients of displacement fields (δu , δv , δw , $\delta\phi_x$, $\delta\phi_y$) in Eqs. (8-9-11) as shown in Eq. (16 - 20):

$$\delta u: \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} - \frac{\partial^2 P_{xz}}{2\partial x\partial y} - \frac{\partial^2 P_{yz}}{2\partial y^2} = I_0\ddot{u} + I_1\ddot{\phi}_x \tag{16}$$

$$\delta v: \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} + \frac{\partial^2 P_{xz}}{2\partial x^2} + \frac{\partial^2 P_{yz}}{2\partial x\partial y} = I_0\ddot{v} + I_1\ddot{\varphi}_y \tag{17}$$

$$\delta w : \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \frac{\partial^2 P_x}{2\partial x \partial y} - \frac{\partial^2 P_y}{2\partial x \partial y} + \frac{\partial^2 P_{xy}}{2\partial y^2} - \frac{\partial^2 P_{xy}}{2\partial x^2} + F_{xy} = I_0 \ddot{w} \qquad (18)$$

$$\delta \phi_x : \frac{\partial M_x}{\partial x} + \partial Q_x + \frac{\partial P_y}{2\partial y} + \frac{\partial P_{xy}}{2\partial x} - \frac{\partial^2 R_{xz}}{2\partial x \partial y} - \frac{\partial^2 R_{yz}}{2\partial y^2} \qquad (19)$$
$$- \frac{\partial P_z}{2\partial y} + \frac{\partial M_{xy}}{\partial y} = I_1 \ddot{u} + I_2 \ddot{\phi}_x$$

$$\delta \phi_y : \frac{\partial M_y}{\partial y} + \partial Q_y + \frac{\partial P_x}{2\partial x} + \frac{\partial P_z}{2\partial x} - \frac{\partial P_{xy}}{2\partial y} + \frac{\partial^2 R_{xz}}{2\partial x^2} \qquad (20)$$
$$+ \frac{\partial^2 R_{yz}}{2\partial x \partial y} + \frac{\partial M_{xy}}{\partial x} = I_1 \ddot{v} + I_2 \ddot{\phi}_y$$

5. Constitutive equations of Nano FG plates

Constitutive equations of Nano FG plates are expressed by Eq. (21)[23]:

$$E(z) = E_m + (E_c - E_m)(0.5 + \frac{z}{h})^n \qquad (21)$$

$$\rho(z) = \rho_m + (\rho_c - \rho_m)(0.5 + \frac{z}{h})^n$$

In Eq. (21): E_m metal elasticity modulus; E_c :ceramic elasticity modulus; n : the power; ρ_m, ρ_c: metal and ceramic density, respectively. rectangular FG plate density and elasticity modulus are in accordance with varying thickness. Eq. (22) describes constitutive linear elastic equations for rectangular FG Nano plate[17]:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{bmatrix} = \frac{E(z)}{1-\upsilon^2} \begin{bmatrix} 1 & \upsilon & 0 & 0 & 0 \\ \upsilon & 1 & 0 & 0 & 0 \\ 0 & 0 & s & 0 & 0 \\ 0 & 0 & 0 & s & 0 \\ 0 & 0 & 0 & 0 & s \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} \qquad (22)$$

Where :

$$s = (1-\upsilon)/2$$

Modified coupling stress is used to employ small-scale parameter in equations. (l) small-scale parameter is attained from the deviatoric section of modified couple stress (Eq. (23)) as [24]:

$$m_{ij} = \frac{l^2 E(z)}{1+\upsilon} \chi_{ij} \qquad (23)$$

Replacing strain equations (Eq. (2)) in linear elastic equation of FGM (Eq.(22)), constitutive equations are written as Eq.(24):

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{bmatrix} = \frac{E(z)}{1-\upsilon^2} \begin{bmatrix} 1 & \upsilon & 0 & 0 & 0 \\ \upsilon & 1 & 0 & 0 & 0 \\ 0 & 0 & s & 0 & 0 \\ 0 & 0 & 0 & s & 0 \\ 0 & 0 & 0 & 0 & s \end{bmatrix} \begin{bmatrix} \varepsilon_x = \frac{\partial u}{\partial x} + z\frac{\partial \phi_x}{\partial x} \\ \varepsilon_y = \frac{\partial v}{\partial y} + z\frac{\partial \phi_y}{\partial y} \\ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + z\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right) \\ \gamma_{xz} = \phi_x + \frac{\partial w}{\partial x} \\ \gamma_{yz} = \phi_y + \frac{\partial w}{\partial y} \end{bmatrix}$$

(24)

Substituting of Eq. (6) in Eq.(23), the deviatoric section of modified couple stress is written as Eq.(25).

$$m_{ij} = \frac{l^2 E(z)}{1+\upsilon} \begin{bmatrix} \frac{1}{2}\left(\frac{\partial^2 w}{\partial x \partial y} - \frac{\partial \phi_y}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial \phi_x}{\partial y} - \frac{\partial^2 w}{\partial x \partial y}\right) \\ \frac{1}{2}\left(\frac{\partial \phi_y}{\partial x} - \frac{\partial \phi_x}{\partial y}\right) \\ \frac{1}{4}\left(\frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w}{\partial x^2} + \frac{\partial \phi_x}{\partial x} - \frac{\partial \phi_y}{\partial y}\right) \\ \frac{1}{4}\left((\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y}) + z(\frac{\partial^2 \phi_y}{\partial x^2} - \frac{\partial^2 \phi_x}{\partial x \partial y})\right) \\ \frac{1}{4}\left((\frac{\partial^2 v}{\partial x \partial y} - \frac{\partial^2 u}{\partial y^2}) + z(\frac{\partial^2 \phi_y}{\partial x \partial y} - \frac{\partial^2 \phi_x}{\partial y^2})\right) \end{bmatrix} \qquad (25)$$

Taking the place of Eqs.(24&25) into Eqs. (12 -15), Stress and modified couple stress resultants expressd as generalize displacement:

$$N_x = \int_{-h/2}^{h/2} \sigma_x dz = \frac{E(z)}{1-\upsilon^2} \int_{-h/2}^{h/2} \left(\varepsilon_x + \upsilon \varepsilon_y\right) dz =$$
$$A \int_{-h/2}^{h/2} \left(\frac{\partial u}{\partial x} + z\frac{\partial \phi_x}{\partial x}\right) + \upsilon\left(\frac{\partial v}{\partial y} + z\frac{\partial \phi_y}{\partial y}\right) dz \qquad (26)$$
$$= A \int_{-h/2}^{h/2} \left(\frac{\partial u}{\partial x} + \upsilon\frac{\partial v}{\partial y}\right) + \int_{-h/2}^{h/2} B\left(\upsilon\frac{\partial \phi_y}{\partial y} + \frac{\partial \phi_x}{\partial x}\right) dz$$

$$N_y = \int_{-h/2}^{h/2} \sigma_y dz = A \int_{-h/2}^{h/2} \left(\varepsilon_y + \upsilon \varepsilon_x\right) dz =$$
$$A \int_{-h/2}^{h/2} (\frac{\partial v}{\partial y} + \upsilon\frac{\partial u}{\partial x}) dz + B \int_{-h/2}^{h/2} \left(\frac{\partial \phi_y}{\partial y} + \upsilon\frac{\partial \phi_x}{\partial x}\right) dz \qquad (27)$$

$$N_{xy} = \int_{-h/2}^{h/2} \sigma_{xy} dz = A\left(\frac{1-\upsilon}{2}\right) \int_{-h/2}^{h/2} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + 2z\frac{\partial^2 w}{\partial x \partial y}\right) dz =$$
$$J_1 \int_{-h/2}^{h/2} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + 2z\frac{\partial^2 w}{\partial x \partial y}\right) dz \qquad (28)$$

$$M_x = \int_{-h/2}^{h/2} \sigma_x z dz = B \int_{-h/2}^{h/2} \left(\frac{\partial u}{\partial x} + z\frac{\partial \phi_x}{\partial x}\right) dz +$$
$$\upsilon \int_{-h/2}^{h/2} \left(\frac{\partial v}{\partial y} + z\frac{\partial \phi_y}{\partial y}\right) dz = \qquad (29)$$
$$B \int_{-h/2}^{h/2} \left(\frac{\partial u}{\partial x} + \upsilon\frac{\partial v}{\partial y}\right) dz + C \int_{-h/2}^{h/2} \left(\frac{\partial \phi_x}{\partial x} + \upsilon\frac{\partial \phi_y}{\partial y}\right) dz$$

$$M_y = \int_{-h/2}^{h/2} \sigma_y z dz = B \int_{-h/2}^{h/2} \left(\frac{\partial v}{\partial y} + \upsilon\frac{\partial u}{\partial x}\right) dz + C \int_{-h/2}^{h/2} \left(\frac{\partial \phi_y}{\partial y} + \upsilon\frac{\partial \phi_x}{\partial x}\right) dz \qquad (30)$$

$$M_{xy} = \int_{-h/2}^{h/2} \sigma_{xy} z dz = \frac{zE(z)}{2(1+\upsilon)} \int_{-h/2}^{h/2} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) dz$$
$$+ \int_{-h/2}^{h/2} \frac{z^2 E(z)}{(1+\upsilon)} \frac{\partial^2 w}{\partial x \partial y} dz \qquad (31)$$
$$= U_1 \int_{-h/2}^{h/2} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) dz + 2L \int_{-h/2}^{h/2} \left(\frac{\partial^2 w}{\partial x \partial y}\right) dz$$

$$Q_x = \int_{-h/2}^{h/2} \sigma_{xz} dz = \frac{E(z)k}{2(1+\upsilon)} \int_{-h/2}^{h/2} \left(\frac{\partial w}{\partial x} + \phi_x \right) dz = O \int_{-h/2}^{h/2} \left(\frac{\partial w}{\partial x} + \phi_x \right) dz \quad (32)$$

$$Q_y = \int_{-h/2}^{h/2} \sigma_{yz} dz = \frac{E(z)k}{2(1+\upsilon)} \int_{-h/2}^{h/2} \left(\frac{\partial w}{\partial y} + \phi_y \right) dz = O \int_{-h/2}^{h/2} \left(\frac{\partial w}{\partial y} + \phi_y \right) dz \quad (33)$$

$$P_x = \int_{-h/2}^{h/2} m_x dz = \frac{l^2 E(z)}{2(1+\upsilon)} \int_{-h/2}^{h/2} \left(\frac{\partial^2 w}{\partial x \partial y} - \frac{\partial \phi_y}{\partial x} \right) dz = D \int_{-h/2}^{h/2} \left(\frac{\partial^2 w}{\partial x \partial y} - \frac{\partial \phi_y}{\partial x} \right) dz \quad (34)$$

$$P_y = \int_{-h/2}^{h/2} m_y dz = D \int_{-h/2}^{h/2} \left(\frac{\partial \phi_x}{\partial y} - \frac{\partial^2 w}{\partial x \partial y} \right) dz \quad (35)$$

$$P_{xz} = \frac{D}{2} \int_{-h/2}^{h/2} \left(\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y} \right) dz + \frac{X}{2} \int_{-h/2}^{h/2} \left(\frac{\partial^2 \phi_y}{\partial x^2} - \frac{\partial^2 \phi_x}{\partial x \partial y} \right) dz \quad (36)$$

$$P_{yz} = \frac{D}{2} \int_{-h/2}^{h/2} \left(\frac{\partial^2 v}{\partial x \partial y} - \frac{\partial^2 u}{\partial y^2} \right) dz + \frac{X}{2} \int_{-h/2}^{h/2} \left(\frac{\partial^2 \phi_y}{\partial x \partial y} - \frac{\partial^2 \phi_x}{\partial y^2} \right) dz \quad (37)$$

$$R_{xz} = \int_{-h/2}^{h/2} z m_{xz} dz = \frac{X}{2} \int_{-h/2}^{h/2} \left(\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y} \right) dz + \frac{F_1}{2} \int_{-h/2}^{h/2} \left(\frac{\partial^2 \phi_y}{\partial x^2} - \frac{\partial^2 \phi_x}{\partial x \partial y} \right) dz \quad (38)$$

$$R_{yz} = \frac{X}{2} \int_{-h/2}^{h/2} \left(\frac{\partial^2 v}{\partial x \partial y} - \frac{\partial^2 u}{\partial y^2} \right) dz + \frac{F_1}{2} \int_{-h/2}^{h/2} \left(\frac{\partial^2 \phi_y}{\partial x \partial y} - \frac{\partial^2 \phi_x}{\partial y^2} \right) dz \quad (39)$$

In Eqs.(26-39), used variable are defined as follows:

$$A = \frac{E(z)}{1-\upsilon^2}, B = \frac{E(z)}{1-\upsilon^2}, C = \frac{E(z)}{1-\upsilon^2},$$
$$J_1 = \frac{E(z)}{2(1+\upsilon)}, X = \frac{l^2 E(z)}{2(1+\upsilon)},$$
$$F_1 = \frac{l^2 E(z)}{2(1+\upsilon)}, U_1 = \frac{E(z)}{2(1+\upsilon)}, L = \frac{E(z)}{2(1+\upsilon)}, \quad (40)$$
$$O = \frac{E(z)k}{2(1+\upsilon)}, D = \frac{l^2 E(z)}{2(1+\upsilon)}$$

6. **Governing equations of motion in terms of displacement** Equations of motion in terms of displacement are defined by using Eqs.(26-39), and replacing in Eqs.(16-20), as

$$\begin{aligned}
\delta u : & A\left(\frac{\partial^2 u}{\partial x^2} + \upsilon \frac{\partial^2 v}{\partial x \partial y} \right) + B\left(\frac{\partial^2 \phi_x}{\partial x^2} + \upsilon \frac{\partial^2 \phi_y}{\partial x \partial y} \right) \\
& + J_1\left(\frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} \right) + 2U_1 \frac{\partial^3 w}{\partial x \partial y^2} \\
& - \frac{D}{4}\left(\frac{\partial^4 v}{\partial y \partial x^3} - \frac{\partial^4 u}{\partial y^2 \partial x^2} \right) - \\
& \frac{X}{4}\left(\frac{\partial^4 \phi_y}{\partial x^3 \partial y} - \frac{\partial^4 \phi_x}{\partial y^2 \partial x^2} \right) - \frac{D}{4}\left(\frac{\partial^4 v}{\partial x \partial y^3} - \frac{\partial^4 u}{\partial y^4} \right) \\
& - \frac{X}{4}\left(\frac{\partial^4 \phi_y}{\partial x \partial y^3} - \frac{\partial^4 \phi_x}{\partial y^4} \right) = I_0 \ddot{u} + I_1 \ddot{\phi}_x
\end{aligned} \quad (41)$$

$$\begin{aligned}
\delta v : & A\left(\frac{\partial^2 v}{\partial y^2} + \upsilon \frac{\partial^2 u}{\partial x \partial y} \right) + B\left(\frac{\partial^2 \phi_y}{\partial y^2} + \upsilon \frac{\partial^2 \phi_x}{\partial x \partial y} \right) \\
& + J_1\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} \right) + 2U_1 \frac{\partial^3 w}{\partial x^2 \partial y} \\
& + \frac{D}{4}\left(\frac{\partial^4 v}{\partial x^4} - \frac{\partial^4 u}{\partial x^3 \partial y} \right) + \frac{X}{4}\left(\frac{\partial^4 \phi_y}{\partial x^4} - \frac{\partial^4 \phi_x}{\partial x^3 \partial y} \right) \\
& + \frac{D}{4}\left(\frac{\partial^4 v}{\partial x^2 \partial y^2} - \frac{\partial^4 u}{\partial x \partial y^3} \right) + \\
& \frac{X}{4}\left(\frac{\partial^4 \phi_y}{\partial x^2 \partial y^2} - \frac{\partial^4 \phi_x}{\partial x \partial y^3} \right) = I_0 \ddot{v} + I_1 \ddot{\phi}_y
\end{aligned} \quad (42)$$

$$\begin{aligned}
\delta w : & 2J_1\left(\frac{\partial \phi_x}{\partial x} + \frac{\partial \phi_y}{\partial y} \right) + \frac{D}{2}\left(\frac{\partial^4 w}{\partial x^2 \partial y^2} - \frac{\partial^3 \phi_y}{\partial x^2 \partial y} \right) \\
& - \frac{D}{2}\left(\frac{\partial^3 \phi_x}{\partial x \partial y^2} - \frac{\partial^4 w}{\partial x^2 \partial y^2} \right) \\
& + \frac{D}{4}\left(\frac{\partial^4 w}{\partial y^4} - \frac{\partial^4 w}{\partial x^2 \partial y^2} - \frac{\partial^3 \phi_y}{\partial y^3} - \frac{\partial^3 \phi_x}{\partial x \partial y^2} \right) \\
& - \frac{D}{4}\left(\frac{\partial^4 w}{\partial x^2 \partial y^2} - \frac{\partial^4 w}{\partial x^4} - \frac{\partial^3 \phi_y}{\partial x^2 \partial y} - \frac{\partial^3 \phi_x}{\partial x^3} \right) \\
& A\left(\frac{\partial Q_x}{\partial x} + \frac{\partial^2 w}{\partial x^2} + \frac{\partial Q_y}{\partial y} + \frac{\partial^2 w}{\partial y^2} \right) + F_{xy} = I_0 \ddot{w}
\end{aligned} \quad (43)$$

$$\begin{aligned}
\delta \phi_x : & B\left(\frac{\partial^2 u}{\partial x^2} + \upsilon \frac{\partial^2 v}{\partial x \partial y} \right) + C\left(\frac{\partial^2 \phi_x}{\partial x^2} + \upsilon \frac{\partial^2 \phi_y}{\partial x \partial y} \right) \\
& + 2J_1\left(\partial Q_x + \frac{\partial w}{\partial x} \right) + \frac{D}{2}\left(\frac{\partial^2 \phi_y}{\partial y \partial x} - \frac{\partial^2 \varphi_x}{\partial y^2} \right) + \\
& \frac{D}{4}\left(\frac{\partial^3 w}{\partial y^2 \partial x} - \frac{\partial^3 w}{\partial x^3} - \frac{\partial^2 \phi_x}{\partial x^2} - \frac{\partial^2 \phi_y}{\partial x \partial y} \right) - \\
& \frac{X}{4}\left(\frac{\partial^4 v}{\partial x \partial y^3} - \frac{\partial^4 u}{\partial y^4} \right) - \frac{F_1}{4}\left(\frac{\partial^4 \phi_y}{\partial x \partial y^3} - \frac{\partial^4 \phi_x}{\partial y^4} \right) - \\
& \frac{X}{4}\left(\frac{\partial^4 v}{\partial x^3 \partial y} - \frac{\partial^4 u}{\partial y^2 \partial x^2} \right) + \frac{F_1}{4}\left(\frac{\partial^4 \phi_y}{\partial x^3 \partial y} - \frac{\partial^4 \phi_x}{\partial y^2 \partial x^2} \right) \\
& + X\left(\frac{\partial^2 \varphi_x}{\partial y^2} - \frac{\partial^3 w}{\partial x \partial y^2} \right) = I_1 \ddot{u} + I_2 \ddot{\phi}_x
\end{aligned} \quad (44)$$

$$\delta\phi_y : B\left(\frac{\partial^2 v}{\partial y^2}+\upsilon\frac{\partial^2 u}{\partial x\,\partial y}\right)+C\left(\frac{\partial^2\phi_y}{\partial y^2}+\upsilon\frac{\partial^2\phi_x}{\partial x\,\partial y}\right)$$
$$+2J_1\left(\partial Q_y+\frac{\partial w}{\partial y}\right)-\frac{D}{2}\left(\frac{\partial^3 w}{\partial x^2\partial y}-\frac{\partial^2\phi_y}{\partial x^2}\right)$$
$$-\frac{D}{2}\left(\frac{\partial^2\varphi_y}{\partial x^2}-\frac{\partial^2\varphi_x}{\partial x\,\partial y}\right)-$$
$$\frac{D}{4}\left(\frac{\partial^3 w}{\partial y^3}-\frac{\partial^3 w}{\partial x^2\partial y}-\frac{\partial^2\phi_x}{\partial x\,\partial y}-\frac{\partial^2\phi_y}{\partial y^2}\right)+$$
$$\frac{X}{4}\left(\frac{\partial^4 v}{\partial x^2\partial y^2}-\frac{\partial^4 u}{\partial x\,\partial y^3}\right) .$$
$$+\frac{F_1}{4}\left(\frac{\partial^4\phi_y}{\partial x^2\partial y^2}-\frac{\partial^4\phi_x}{\partial y^3\partial x}\right)+$$
$$\frac{X}{4}\left(\frac{\partial^4 v}{\partial x^4}-\frac{\partial^4 u}{\partial y\,\partial x^3}\right)+\frac{F_1}{4}\left(\frac{\partial^4\phi_y}{\partial x^4}-\frac{\partial^4\phi_x}{\partial y\,\partial x^3}\right)$$
$$=I_1\ddot{v}+I_2\ddot{\phi}_y \tag{45}$$

7. Boundary conditions:

Consider a simply supported Nano rectangular FG plate by considering length a, width b and thickness h, which is imposed a moving load. Boundary conditions for the plate indicated as fig(2):

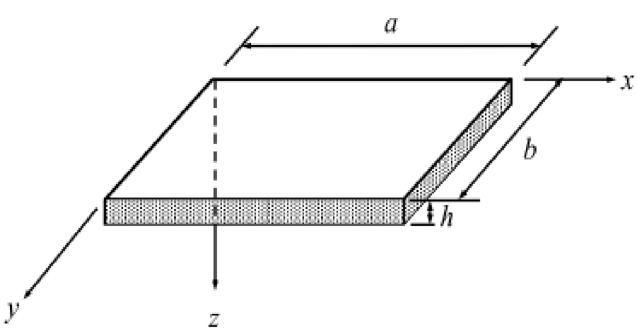

Figure 2. The geometry of a Nano rectangular FG plate[25]

$$u(x,0)=u(x,b)=0 \qquad w(0,y)=w(a,y)=0$$
$$\phi_x(x,0)=\phi_x(x,b)=0 \qquad w(x,0)=w(x,b)=0$$
$$\phi_y(0,y)=\phi_y(a,y)=0 \qquad v(0,y)=v(a,y)=0$$

. (46)

Dimensionless relations

Considering the following relations, the dimensionless equations will be obtained [8].

$$U_2=\frac{u}{h},\ V=\frac{v}{h},\ W=\frac{w}{h}X=\frac{x}{h},$$
$$Y=\frac{y}{h},,\ \varpi=\frac{\omega a^2}{h}\sqrt{\frac{\rho_c}{E_c}} \tag{47}$$

8. Analytical Solution

Navier solution for simply supported nano rectangular FG plate is employed to solve Nano FGM equations. according to FSDT Eq.(48) is as follows [20]:

$$U(X,Y,T)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}U_{mn}\cos(\alpha X)\sin(\beta Y)e^{i\varpi T}$$
$$V(X,Y,T)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}V_{mn}\sin(\alpha X)\cos(\beta Y)e^{i\varpi T} \tag{48}$$
$$W(X,Y,T)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}\sin(\alpha X)\sin(\beta Y)e^{i\varpi T}$$
$$\phi_x(X,Y,T)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}X_{mn}\cos(\alpha X)\sin(\beta Y)e^{i\varpi T}$$
$$\phi_y(X,Y,T)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}Y_{mn}\sin(\alpha X)\cos(\beta Y)e^{i\varpi T}$$

In Eq.(48), the parameters are as Eq.(49)[20]:

$$\alpha=\frac{m\pi}{a}\ \text{and}\ i=\sqrt{-1}\ \text{and}\ \beta=\frac{n\pi}{b} \tag{49}$$

$\omega, U, V, W, \phi_x, \phi_y$ are frequency and deflection field coefficients of Eq.(48) in dual series of analytical solution of the Navier. The double trigonometric series for load is:

$$q(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}Q_{mn}\sin\alpha x\sin\beta y \tag{50}$$

Where

$$Q_{mn}=\frac{4}{ab}\int_0^a\int_0^b q(x,y)\sin\alpha x\sin\beta y\,dxdy$$

. (51)

In which

$$\begin{bmatrix} q_0 & \textit{for sinusoidal load} \\ \dfrac{16}{q_0 m n\pi^2} & \textit{for uniform load}\end{bmatrix} \tag{52}$$

Replacing Eqs.(48&50) into dimensionless equations and simplification, matrix form of equations are as:

(53)

$$\left(\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15}\\ s_{21} & s_{22} & s_{23} & s_{24} & s_{25}\\ s_{31} & s_{32} & s_{33} & s_{34} & s_{35}\\ s_{41} & s_{42} & s_{43} & s_{44} & s_{45}\\ s_{51} & s_{52} & s_{53} & s_{54} & s_{55}\end{bmatrix}-\omega^2\begin{bmatrix} I_0 & 0 & 0 & I_1 & 0\\ 0 & I_0 & 0 & I_1 & I_1\\ 0 & 0 & I_0 & 0 & 0\\ I_1 & I_1 & 0 & I_2 & 0\\ 0 & I_1 & 0 & 0 & I_2\end{bmatrix}\right)\begin{pmatrix} U_{mn}\\ V_{mn}\\ W_{mn}\\ X_{mn}\\ Y_{mn}\end{pmatrix}=\begin{bmatrix}0\\0\\F_{xy}\\0\\0\end{bmatrix}$$

$$S_{11} = A_1\alpha^2 + J_{11}\beta^2 + \frac{D_1}{2}\beta^2(\alpha^2+1) + X_1\beta^2$$
$$S_{12} = A_1\upsilon\alpha\beta + J_{11}\beta\alpha + \frac{D_1}{2}\left(\alpha\beta^3 + \beta\alpha^3\right)$$
$$S_{13} = U_{11}\alpha\beta$$
$$S_{14} = B_1\alpha^2 + X_1\beta^2\left(\alpha^2+\beta^2\right)$$
$$S_{15} = \upsilon\alpha\beta B_1 - X_1\alpha^3\beta + X_1\alpha\beta^3$$
$$S_{22} = A_1\beta^2 + J_{11}A\alpha^2 + \frac{D_1}{2}\alpha^2(\alpha+\beta^2)$$
$$S_{23} = U_{11}\alpha^2\beta$$
$$S_{24} = B_1\upsilon\alpha\beta + X_1\left(\alpha\beta^3+\beta\alpha^3\right)$$
$$S_{25} = B_1\beta^2 + X_1\left(\alpha^2\beta^2+\alpha^4\right)$$
$$S_{33} = 2L_1\alpha^2\beta^2 + 2J_{11}\left(\alpha^2+\beta^2\right) + \frac{D_1}{2}\left(3\alpha^2\beta^2+\beta^3-\alpha^4+\beta^4\right)$$
$$S_{34} = 2J_{11}\alpha + \frac{D_1}{2}\left(2\alpha\beta^2+\alpha^3\right)$$
$$S_{35} = 2J_{11}\beta + \frac{D_1}{2}\left(\beta^3+2\alpha^2\beta\right)$$
$$S_{44} = B_1\alpha^2 + 2J_{11} + \frac{D_1}{2}\left(\alpha^2+\beta^2\right) + \frac{X_1}{2}\left(\beta^3+\alpha^2\beta^2\right) + \frac{F_{11}}{2}\beta^2$$
$$S_{45} = B_1\upsilon\alpha\beta + D_1\alpha\beta + \frac{F_{11}}{2}\alpha\beta^3 + \frac{F_{11}}{2}\alpha^3\beta$$
$$S_{55} = C_1\beta + D_1\alpha^2 + 2J_{11} + \frac{F_{11}}{2}\left(\alpha^2\beta^2+\alpha^4\right)$$

It should be noticed that some aforementioned variable are:

$$A_1 = \int_{-h/2}^{h/2} A\,dz,\ B_1 = zA_1, C_1 = z^2A_1, J_{11} = \int_{-h/2}^{h/2} J_1\,dz,\ U_{11} = z\,J_{11}, L_1 = zU_{11},\quad O_1 = \int_{-h/2}^{h/2} O\,dz,\ D_1 = \int_{-h/2}^{h/2} D\,dz,\ X_1 = zD_1, F_{11} = zX_1 \tag{54}$$

Section 2: simple FSDT equations

11 - Displacement fields

Conventional FSDT displacement fields are expressed as [26]:

$$\begin{aligned} u_1(x,y,z,t) &= u(x,y,t) + z\,\phi_x \\ u_2(x,y,z,t) &= v(x,y,t) + z\,\phi_y \\ u_3(x,y,z,t) &= w(x,y,t) \end{aligned} \tag{55}$$

Where u, v and w are displacement alongside of x, y and z coordinate directions of a point on the midplane of the plate. By making further assumptions and dividing transverse displacement w into bending and shear parts ($w=w_b+w_s$) the 5 unknown displacement function of the midplane plate's, reduced to 4 parameters. Shear normal vectors are defined as: $\varphi_x = -\frac{\partial w_b}{\partial x}, \varphi_y = -\frac{\partial w_b}{\partial y}$, so that the new simple displacement fields can be rewritten[16]:

$$\begin{aligned} u_1(x,y,z) &= u(x,y) - z\,\frac{\partial w_b}{\partial x} \\ u_2(x,y,z) &= v(x,y) - z\,\frac{\partial w_b}{\partial y} \\ u_3(x,y,z) &= w_b(x,y) + w_s(x,y) \end{aligned} \tag{56}$$

The simple first order shear deformation theory associated strains are:

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} - z\,\frac{\partial^2 w_b}{\partial x^2} \\ \varepsilon_y &= \frac{\partial v}{\partial y} - z\,\frac{\partial^2 w_b}{\partial y^2} \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2z\,\frac{\partial^2 w_b}{\partial x\,\partial y} \\ \gamma_{xz} &= \frac{\partial w_s}{\partial x} \quad \gamma_{yz} = \frac{\partial w_s}{\partial y} \end{aligned} \tag{57}$$

12. Modified Couple Stress Theory

In order to strain energy extraction, modified couple stress theory is used, which it has only one length scale parameter. Regarding the theory, virtual strain energy written as[1]:

$$\delta U = \int_V \sigma_{ij}\,\delta\varepsilon_{ij}\,dV + \int_v m_{ij}\,\delta\chi_{ij}\,dV \tag{58}$$

The parameters of Eq.(58) already introduced in Eq.(10). General function curvature form, based on rotation vectors present as follows[20]:

$$\chi = \frac{1}{2}\left[\nabla\omega + \left(\nabla\omega\right)^T\right] \quad , \ \omega = \frac{1}{2}\nabla\times u \tag{59}$$

$$\chi_{ij} = \frac{1}{2}\left(\frac{\partial\theta_i}{\partial x_j} + \frac{\partial\theta_j}{\partial x_i}\right)$$
$$i, j = 1,2,3$$

Curvature functions as displacement fields are[3]:

$$\begin{aligned} \theta_x &= \theta_1 = \frac{1}{2}\left(\frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3}\right) \\ \theta_y &= \theta_2 = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) \\ \theta_z &= \theta_3 = \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2}\right) \end{aligned} \tag{60}$$

By replacing of displacement fields i.e., Eq.(56) into Eq.(60), rotation vectors in terms of displacement fields are rewritten as in Eq.(61).

$$\theta_x = \frac{1}{2}\left(\frac{2\partial w_b}{\partial y} + \frac{\partial w_s}{\partial y}\right)$$
$$\theta_y = -\frac{1}{2}\left(\frac{2\partial w_b}{\partial x} + \frac{\partial w_s}{\partial x}\right) \tag{61}$$
$$\theta_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

Curvature function in terms of displacement fields are attained by replacing of Eq.(56) into Eq.(59) as:

$$\chi_{xx} = \frac{1}{2}\left(\frac{2\partial^2 w_b}{\partial x\,\partial y} + \frac{\partial^2 w_s}{\partial x\,\partial y}\right)$$
$$\chi_{yy} = -\frac{1}{2}\left(\frac{2\partial^2 w_b}{\partial x\,\partial y} + \frac{\partial^2 w_s}{\partial x\,\partial y}\right)$$
$$\chi_{xy} = \frac{1}{4}\left(\frac{2\partial^2 w_b}{\partial y^2} + \frac{\partial^2 w_s}{\partial y^2} - \frac{\partial^2 w_s}{\partial x^2} - \frac{2\partial^2 w_b}{\partial x^2}\right) \tag{62}$$
$$\chi_{xz} = \frac{1}{4}\left(-\frac{\partial^2 u}{\partial x\,\partial y} + \frac{\partial^2 v}{\partial x^2}\right)$$
$$\chi_{yz} = \frac{1}{4}\left(-\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x\,\partial y}\right)$$

13. Equations of motion

Equations of motion derived from Hamilton's principle are defined as[25]:

$$\int_0^T (\delta U + \delta V - \delta K)\,dt \tag{63}$$

In which: δU are virtual strains energy, δW are virtual work done by external force, δK are kinetic energy. δW Which is a moving load, assumed as [21]:

$$\delta W = \int F\,\delta(\mathrm{x}-\mathrm{x}_0(\mathrm{t}))\delta(y - y_0(\mathrm{t}))dA\,dw = \int F\delta(\mathrm{x}-\mathrm{x}_0)\delta(y - y_0)dA\,dw \tag{64}$$

Kinetic energy can be obtained as[17]:

$$\delta K = \int_A \int_{-h/2}^{h/2} \rho(z)\left(\dot u_1\delta\dot u_1 + \dot u_2\delta\dot u_2 + \dot u\,\delta\dot u_3\right)dA\,dz$$
$$= \int_A I_0\left(\dot u\,\delta\dot u + \dot v\,\delta\dot v + (\dot w_b + \dot w_s)\delta(\dot w_b + \dot w_s)\right)$$
$$+ I_2\left(\frac{\partial\dot w_b}{\partial x}\frac{\partial\delta\dot w_b}{\partial x} + \frac{\partial\dot w_b}{\partial y}\frac{\delta\,\partial\dot w_b}{\partial y}\right) - \tag{65}$$
$$I_1\left(\dot u\,\frac{\partial\delta\dot w_b}{\partial x} + \frac{\partial\dot w_b}{\partial x}\delta\dot u + \dot v\,\frac{\partial\delta\dot w_b}{\partial y} + \delta\dot v\,\frac{\partial\dot w_b}{\partial y}\right)$$

Shear strains energy according to modified couple stress theory can be expressed as:

$$\delta\mathrm{U} = (\int_{\mathrm{A}}\int_{-\mathrm{h}/2}^{\mathrm{h}/2} \sigma_x\delta\varepsilon_x + \sigma_y\delta\varepsilon_y + \sigma_z\delta\varepsilon_z + \sigma_{xy}\delta\gamma_{xy} + \sigma_{xz}\delta\gamma_{xz} + \sigma_{yz}\delta\gamma_{yz} + \mathrm{m}_x\delta\chi_{xx}$$
$$\mathrm{m}_y\delta\chi_{yy} + \mathrm{m}_z\delta\chi_{zz} + 2\mathrm{m}_{xy}\delta\chi_{xy} + 2\mathrm{m}_{xz}\delta\chi_{xz} + 2\mathrm{m}_{yz}\delta\chi_{yz})\mathrm{d}A\mathrm{d}z =$$
$$\delta\int_{-\mathrm{h}/2}^{\mathrm{h}/2} \sigma_x\left(\frac{\partial\mathrm{u}}{\partial\mathrm{x}} - \mathrm{z}\frac{\partial^2\mathrm{w_b}}{\partial\mathrm{x}^2}\right) + \sigma_y\left(\frac{\partial\mathrm{v}}{\partial\mathrm{y}} - \mathrm{z}\frac{\partial^2\mathrm{w_b}}{\partial\mathrm{y}^2}\right)$$
$$+\sigma_{xy}\left(\frac{\partial\mathrm{u}}{\partial\mathrm{y}} + \frac{\partial\mathrm{v}}{\partial\mathrm{x}} - 2\mathrm{z}\frac{\partial^2\mathrm{w_b}}{\partial\mathrm{x}\partial\mathrm{y}}\right) +$$
$$\sigma_{xz}\left(\frac{\partial\mathrm{w_s}}{\partial\mathrm{x}}\right) + \sigma_{yz}\left(\frac{\partial\mathrm{w_s}}{\partial\mathrm{x}}\right) + \mathrm{m}_x\frac{1}{2}\left(\frac{\partial^2\mathrm{w_b}}{\partial\mathrm{x}\partial\mathrm{y}} + \frac{\partial^2\mathrm{w_s}}{2\partial\mathrm{x}\partial\mathrm{y}}\right)$$
$$-\mathrm{m}_y\frac{1}{2}\left(\frac{\partial^2\mathrm{w_b}}{\partial\mathrm{x}\partial\mathrm{y}} + \frac{\partial^2\mathrm{w_s}}{2\partial\mathrm{x}\partial\mathrm{y}}\right) +$$
$$\mathrm{m}_{xy}\frac{1}{4}\left(\frac{2\partial^2\mathrm{w_b}}{\partial\mathrm{y}^2} + \frac{\partial^2\mathrm{w_s}}{\partial\mathrm{y}^2} - \frac{2\partial^2\mathrm{w_b}}{\partial\mathrm{x}^2} - \frac{\partial^2\mathrm{w_s}}{\partial\mathrm{x}^2}\right)$$
$$+\frac{1}{4}\mathrm{m}_{xz}\left(\frac{\partial^2\mathrm{v}}{\partial\mathrm{x}^2} - \frac{\partial^2\mathrm{u}}{\partial\mathrm{y}\partial\mathrm{x}}\right) +$$

$$\frac{1}{4}m_{yz}\left(\frac{\partial^2 v}{\partial x\,\partial y} - \frac{\partial^2 u}{\partial y^2}\right) =$$
$$N_x\delta\frac{\partial u}{\partial x} - M_x\delta\frac{\partial^2 w_b}{\partial x^2} + N_y\delta\frac{\partial v}{\partial y} -$$
$$M_y\delta\frac{\partial^2 w_b}{\partial y^2} + N_{xy}\delta\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) -$$
$$2M_{xy}\delta\left(\frac{\partial^2 w_b}{\partial x\,\partial y}\right) + Q_x\delta\frac{\partial w_s}{\partial x}$$
$$+Q_y\frac{\delta\partial w_s}{\partial y} + P_x\frac{\delta}{2}\left(\frac{\partial^2 w_b}{\partial x\,\partial y} + \frac{\partial^2 w_s}{2\partial x\,\partial y}\right)$$
$$+P_y\frac{\delta}{2}\left(\frac{\partial^2 w_b}{\partial x\,\partial y} + \frac{\partial^2 w_s}{2\partial x\,\partial y}\right)$$
$$+\frac{\delta}{4}P_{xz}\left(-\frac{\partial^2 u}{\partial y\,\partial x} + \frac{\partial^2 v}{\partial x^2}\right) +$$
$$P_{xy}\frac{\delta}{4}\left(\frac{2\partial^2 w_b}{\partial y^2} + \frac{\partial^2 w_s}{\partial y^2} - \frac{2\partial^2 w_b}{\partial x^2} - \frac{\partial^2 w_s}{\partial x^2}\right)$$
$$+\frac{1}{4}P_{yz}\delta\left(-\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x\,\partial y}\right) \tag{66}$$

To simplify equation of 12, stress resultants are expressed as: $I_n = \int_{-h/2}^{h/2} \rho(z^n)dz \quad n = 0,1,2$

. (67)

$$R_i = \int_{-h/2}^{h/2} z m_i dz$$
$$Q_i = k \int_{-h/2}^{h/2} \sigma_i dz \tag{68}$$
$$i = xz, yz \quad k = 5/6$$

$$N_j = \int_{-h/2}^{h/2} \sigma_j dz$$
$$M_j = \int_{-h/2}^{h/2} \sigma_j z dz \tag{69}$$
$$j = x, y, xy$$

$$P_k = \int_{-h/2}^{h/2} m_k dz \tag{70}$$
$$k = x, y, xy, yz, xz$$

By replacing of Eqs.(64-66) into Eq.(63), integrating by-parts and then gathering coefficient of displacement fields ($\delta u, \delta v, \delta w_b, \delta w_s$) following equations are obtained.

$$\delta u : \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} - \frac{\partial^2 P_{xz}}{4\partial x \partial y} - \frac{\partial^2 P_{yz}}{4\partial y^2} = I_0 \ddot{u} - I_1 \frac{\partial \ddot{w}_b}{\partial x}$$
$$\delta v : \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} + \frac{\partial^2 P_{xz}}{4\partial x^2} + \frac{\partial^2 P_{yz}}{4\partial y \partial x} = I_0 \ddot{v} - I_1 \frac{\partial \ddot{w}_b}{\partial y}$$
$$\delta w_b : \frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_y}{\partial y^2} - 2\frac{\partial^2 M_{xy}}{\partial y \partial x} + \frac{1}{2}\left(\frac{\partial^2 P_x}{\partial x \partial y} + \frac{\partial^2 P_y}{\partial y \partial x}\right) -$$
$$\frac{1}{2}\left(\frac{\partial^2 P_{yx}}{\partial x^2} - \frac{\partial^2 P_{xy}}{\partial y^2}\right) + F_{xy} = I_0(\ddot{w}_b + \ddot{w}_s) +$$
$$I_1\left(\frac{\partial \ddot{u}}{\partial x} + \frac{\partial \ddot{v}}{\partial y}\right) - I_2 \nabla^2 \ddot{w}_b$$
$$\delta w_s : \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \frac{\partial^2 P_x}{4\partial y \partial x} + \frac{\partial^2 P_y}{4\partial y \partial x} +$$
$$\frac{1}{2}\left(\frac{\partial^2 P_{xy}}{\partial y^2} - \frac{\partial^2 P_{yx}}{\partial x^2}\right) + F_{xy} = I_0(\ddot{w}_b + \ddot{w}_s) \tag{71}$$

14. Constitutive Nano FG relations

Regard a FG plate as shown in fig.(1), the material properties of the plate such as Young's modulus, mass density, are presumed varying through the thickness by power law as[27]:

$$E(z) = E_m + (E_c - E_m)(0.5 + \frac{z}{h})^n \tag{72}$$
$$\rho(z) = \rho_m + (\rho_c - \rho_m)(0.5 + \frac{z}{h})^n$$

Eq.(72) elements are as FSDT, which is already presented. The linear constitutive relations of FG plate are:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{bmatrix} = \frac{E(z)}{1-\upsilon^2} \begin{bmatrix} 1 & \upsilon & 0 & 0 & 0 \\ \upsilon & 1 & 0 & 0 & 0 \\ 0 & 0 & s & 0 & 0 \\ 0 & 0 & 0 & s & 0 \\ 0 & 0 & 0 & 0 & s \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} \tag{73}$$

Where:

$$s = (1-\upsilon)/2$$

Material length scale parameter which is considered as a material property measuring the effect of modified couple stress expressed[24]:

$$m_{ij} = \frac{l^2 E(z)}{1+\upsilon} \chi_{ij} \tag{74}$$

Substituting of Eqs.(57 and 62) into Eqs.(73 and 74) generalized linear constitutive equations can be written as Eqs.(75&76).

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{bmatrix} = \frac{E(z)}{1-\upsilon^2} \begin{bmatrix} 1 & \upsilon & 0 & 0 & 0 \\ \upsilon & 1 & 0 & 0 & 0 \\ 0 & 0 & s & 0 & 0 \\ 0 & 0 & 0 & s & 0 \\ 0 & 0 & 0 & 0 & s \end{bmatrix} \begin{bmatrix} \varepsilon_x = \frac{\partial u}{\partial x} - z\frac{\partial^2 w_b}{\partial x^2} \\ \varepsilon_y = \frac{\partial v}{\partial y} - z\frac{\partial^2 w_b}{\partial y^2} \\ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2z\frac{\partial^2 w_b}{\partial x \partial y} \\ \gamma_{xz} = \frac{\partial w_s}{\partial x} \\ \gamma_{yz} = \frac{\partial w_s}{\partial y} \end{bmatrix} \tag{75}$$

$$m_{ij} = \frac{l^2 E(z)}{1+\upsilon} \begin{bmatrix} \chi_{xx} = \frac{1}{2}\left(\frac{2\partial^2 w_b}{\partial x \partial y} + \frac{\partial^2 w_s}{\partial x \partial y}\right) \\ \chi_{yy} = -\frac{1}{2}\left(\frac{2\partial^2 w_b}{\partial x \partial y} + \frac{\partial^2 w_s}{\partial x \partial y}\right) \\ \chi_{xy} = \frac{1}{4}\left(\frac{2\partial^2 w_b}{\partial y^2} + \frac{\partial^2 w_s}{\partial y^2} - \frac{\partial^2 w_s}{\partial x^2} - \frac{2\partial^2 w_b}{\partial x^2}\right) \\ \chi_{xz} = \frac{1}{4}\left(\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y}\right) \\ \chi_{yz} = \frac{1}{4}\left(\frac{\partial^2 v}{\partial x \partial y} - \frac{\partial^2 u}{\partial y^2}\right) \end{bmatrix} \tag{76}$$

Replacing of Eqs.(75 and 76) into Eqs.(67-70), stress resultants rewritten as displacement fields that are stated as bellows:

$$N_x = \frac{E(z)}{1-\upsilon^2} \int_{-h/2}^{h/2} \left(\frac{\partial u}{\partial x} - z\frac{\partial^2 w_b}{\partial x^2}\right) + \upsilon\left(\frac{\partial v}{\partial y} - z\frac{\partial^2 w_b}{\partial y^2}\right) dz$$
$$= A_1\left(\frac{\partial u}{\partial x} + \upsilon\frac{\partial v}{\partial y}\right) - B_1\left(\frac{\partial^2 w_b}{\partial x^2} + \upsilon\frac{\partial^2 w_b}{\partial y^2}\right) \tag{77}$$

$$N_y = \frac{E(z)}{1-\upsilon^2} \int_{-h/2}^{h/2} \left(\frac{\partial v}{\partial y} - z\frac{\partial^2 w_b}{\partial y^2}\right) +$$
$$\upsilon\left(\frac{\partial u}{\partial x} - z\frac{\partial^2 w_b}{\partial x^2}\right) dz = A_1\left(\frac{\partial v}{\partial y} + \upsilon\frac{\partial u}{\partial x}\right) \tag{78}$$
$$-B_1\left(\frac{\partial^2 w_b}{\partial y^2} + \upsilon\frac{\partial^2 w_b}{\partial x^2}\right)$$

$$N_{xy} = \frac{E(z)}{2(1+\upsilon)} \int_{-h/2}^{h/2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2z \frac{\partial^2 w_b}{\partial y \partial x} \right) dz$$
$$= C_1 \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) - 2D_1 \left(\frac{\partial^2 w_b}{\partial y \partial x} \right) \quad (79)$$

$$M_y = \frac{E(z)}{1-\upsilon^2} \int_{-h/2}^{h/2} z \left(\frac{\partial v}{\partial y} - z \frac{\partial^2 w_b}{\partial x^2} \right) + \upsilon z \left(\frac{\partial u}{\partial x} - z \frac{\partial^2 w_b}{\partial y^2} \right) dz$$
$$= N_1 \left(\frac{\partial v}{\partial y} + \upsilon \frac{\partial u}{\partial x} \right) - T_1 \left(\frac{\partial^2 w_b}{\partial x^2} + \upsilon \frac{\partial^2 w_b}{\partial y^2} \right) \quad (80)$$

$$M_x = \frac{E(z)}{1-\upsilon^2} \int_{-h/2}^{h/2} z \left(\frac{\partial u}{\partial x} - z \frac{\partial^2 w_b}{\partial x^2} \right) +$$
$$\upsilon z \left(\frac{\partial v}{\partial y} - z \frac{\partial^2 w_b}{\partial y^2} \right) dz = N_1 \left(\frac{\partial u}{\partial x} + \upsilon \frac{\partial v}{\partial y} \right) \quad (81)$$
$$-T_1 \left(\frac{\partial^2 w_b}{\partial x^2} + \upsilon \frac{\partial^2 w_b}{\partial y^2} \right)$$

$$M_{xy} = \frac{E(z)}{2(1+\upsilon)} \int_{-h/2}^{h/2} z \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} - z \frac{\partial^2 w_b}{\partial x \partial y} \right) dz$$
$$= D_1 \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) - h_1 \left(\frac{\partial^2 w_b}{\partial x \partial y} \right) \quad (82)$$

$$P_x = \int_{-h/2}^{h/2} m_x dz = \int_{-h/2}^{h/2} \frac{l^2 E(z)}{2(1+\upsilon)} \left(\frac{2\partial^2 w_b}{\partial y \partial x} + \frac{\partial^2 w_s}{\partial y \partial x} \right) dz$$
$$= U_3 \left(\frac{2\partial^2 w_b}{\partial y \partial x} + \frac{\partial^2 w_s}{2\partial y \partial x} \right) \quad (83)$$

$$P_y = \int_{-h/2}^{h/2} m_y dz = -\int_{-h/2}^{h/2} \frac{l^2 E(z)}{2(1+\upsilon)} \left(\frac{2\partial^2 w_b}{\partial y \partial x} + \frac{\partial^2 w_s}{\partial y \partial x} \right) dz$$
$$= U_3 \left(\frac{2\partial^2 w_b}{\partial y \partial x} + \frac{\partial^2 w_s}{2\partial y \partial x} \right) \quad (84)$$

$$P_{xy} = \int_{-h/2}^{h/2} m_{xy} dz = \int_{-h/2}^{h/2} \frac{l^2 E(z)}{4(1+\upsilon)} \left(\frac{2\partial^2 w_b}{\partial y^2} - \frac{2\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_s}{\partial y^2} - \frac{\partial^2 w_s}{\partial x^2} \right) dz =$$
$$U_3 \left(\frac{\partial^2 w_b}{2\partial y^2} - \frac{\partial^2 w_b}{2\partial x^2} + \frac{\partial^2 w_s}{4\partial y^2} - \frac{\partial^2 w_s}{4\partial x^2} \right) \quad (85)$$

$$P_{xz} = \int_{-h/2}^{h/2} m_{xz} dz = \int_{-h/2}^{h/2} \frac{l^2 E(z)}{4(1+\upsilon)} \left(\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 u}{\partial y \partial x} \right) dz$$
$$= U_3 \frac{1}{2} \left(\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 u}{\partial y \partial x} \right) \quad (86)$$

$$P_{yz} = \int_{-h/2}^{h/2} m_{yz} dz = \int_{-h/2}^{h/2} \frac{l^2 E(z)}{4(1+\upsilon)} \left(\frac{\partial^2 v}{\partial x \partial y} - \frac{\partial^2 u}{\partial y^2} \right) dz$$
$$= U_3 \frac{1}{2} \left(\frac{\partial^2 v}{\partial x \partial y} - \frac{\partial^2 u}{\partial y^2} \right) \quad (87)$$

The parameters which are Aforementioned are:

$$A_1 = \int_{-h/2}^{h/2} \frac{E(z)}{1-\upsilon^2} dz \,,\; B_1 = \int_{-h/2}^{h/2} \frac{E(z)}{1-\upsilon^2} z dz \,,$$
$$C_1 = \int_{-h/2}^{h/2} \frac{E(z)}{2(1+\upsilon)} dz \,,\; h_1 = \int_{-h/2}^{h/2} \frac{z^2 E(z)}{2(1+\upsilon)} dz \,,$$
$$D_1 = \int_{-h/2}^{h/2} \frac{zE(z)}{2(1+\upsilon)} dz \,,\; N_1 = B_1, \quad (88)$$
$$T_1 = \int_{-h/2}^{h/2} \frac{E(z)}{1-\upsilon^2} z^2 dz \,,\; U_3 = \frac{l^2 E(z)}{2(1+\upsilon)} \int dz$$

Equations of motion in terms of displacement were obtained by replacing of Eqs.(77-87) into Eqs.(71), as shown in coming equations:

$$\delta u : \left(A_1 \left(\frac{\partial^2 u}{\partial x^2} + \upsilon \frac{\partial^2 v}{\partial y \partial x} \right) - B_1 \left(\frac{\partial^3 w_b}{\partial x^3} + \upsilon \frac{\partial^3 w_b}{\partial y^2 \partial x} \right) \right)$$
$$+ \left(C_1 \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y} \right) - 2D_1 \frac{\partial^3 w_b}{\partial y^2 \partial x} \right)$$
$$-U_3 \left(\frac{\partial^4 u}{4\partial x^2 \partial y^2} - \frac{\partial^4 v}{4\partial y \partial x^3} + \frac{\partial^4 v}{4\partial y^3 \partial x} + \frac{\partial^4 u}{4\partial y^4} \right)$$
$$= I_0 \ddot{u} - I_1 \frac{\partial \ddot{w}_b}{\partial x} \quad (89)$$

$$\delta v : \left(A_1 \left(\frac{\partial^2 v}{\partial y^2} + \upsilon \frac{\partial^2 u}{\partial y \partial x} \right) - B_1 \left(\frac{\partial^3 w_b}{\partial y^3} + \upsilon \frac{\partial^3 w_b}{\partial x^2 \partial y} \right) \right)$$
$$+ \left(C_1 \left(\frac{\partial^2 u}{\partial y \partial x} + \frac{\partial^2 v}{\partial x^2} \right) \right) - 2D_1 \frac{\partial^3 w_b}{\partial x^2 \partial y} +$$
$$U_3 \left(-\frac{\partial^4 u}{4\partial x^3 \partial y} + \frac{\partial^4 v}{4\partial x^4} + \frac{\partial^4 v}{4\partial y^2 \partial x^2} - \frac{\partial^4 u}{4\partial y^3 \partial x} \right) \quad (90)$$
$$= I_0 \ddot{v} - I_1 \frac{\partial \ddot{w}_b}{\partial y}$$

$$\delta w_s : \frac{1}{2} U_3 \left(\frac{\partial^4 w_b}{2\partial x^2 \partial y^2} + \frac{\partial^4 w_s}{8\partial x^2 \partial y^2} + \frac{\partial^4 w_b}{2\partial y^4} \right)$$
$$U_3 \left(+ \frac{\partial^4 w_s}{4\partial y^4} - \frac{\partial^4 w_s}{4x^4} + \frac{\partial^4 w_b}{2x^4} \right) \quad (91)$$
$$C_1 \frac{\partial^2 w_s}{\partial x^2} + C_1 \frac{\partial^2 w_s}{\partial y^2} + F_{xy} = I_0 (\ddot{w}_b + \ddot{w}_s)$$

$$\delta w_b : N_1 \left(\frac{\partial^3 u}{\partial x^3} + \upsilon \frac{\partial^3 v}{\partial x^2 \partial y} \right) - T_1 \left(\frac{\partial^4 w_b}{\partial x^4} + \frac{\partial^4 w_b}{\partial x^2 \partial y^2} \right)$$
$$+ N_1 \left(\frac{\partial^3 v}{\partial y^3} + \upsilon \frac{\partial^3 u}{\partial x \partial y^2} \right) - T_1 \left(\frac{\partial^4 w_b}{\partial x^2 \partial y^2} + \frac{\partial^4 w_b}{\partial y^4} \right) -$$
$$2D_1 \left(\frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial^3 v}{\partial x^2 \partial y} \right) - h_1 \frac{\partial^4 w_b}{\partial x^2 \partial y^2} +$$
$$U_3 \left(\frac{5\partial^4 w_b}{2\partial x^2 \partial y^2} + \frac{3\partial^4 w_s}{4\partial x^2 \partial y^2} + \frac{\partial^4 w_s}{8\partial y^4} + \frac{\partial^4 w_b}{4\partial y^4} + \frac{\partial^4 w_s}{8\partial x^4} + \frac{\partial^4 w_b}{4\partial x^4} \right)$$
$$+ F_{xy} = I_0 (\ddot{w}_b + \ddot{w}_s) + I_1 \left(\frac{\partial \ddot{u}}{\partial x} + \frac{\partial \ddot{v}}{\partial y} \right) - I_2 \nabla^2 \ddot{w}_b$$

. (92)

Boundary conditions and dimensionless equations for SFSDT also stated as FSDT, which are already represented.

15. Analytical solution

Consider a simply supported FG plate with length a width b and thickness h under moving load, as in fig.(1). Solution based on Navier approach can be represented as[17]:

$$\begin{aligned} U(X,Y,T) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} U_{mn}\cos(\alpha X)\sin(\beta Y)e^{i\varpi T} \\ V(X,Y,T) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} V_{mn}\sin(\alpha X)\cos(\beta Y)e^{i\varpi T} \\ W_b(X,Y,T) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{bmn}\sin(\alpha X)\sin(\beta Y)e^{i\varpi T} \\ W_s(X,Y,T) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{smn}\sin(\alpha X)\sin(\beta Y)e^{i\varpi T} \end{aligned} \quad (93)$$

Where: $i=\sqrt{1}$, $\alpha=\dfrac{m\pi}{a}$, $\beta=\dfrac{n\pi}{b}$, $U_{mn}, V_{mn}, W_{bmn}, W_{smn}$ are coefficients, and ω is the frequency of free vibration.

By replacing of Eqs.(50&93) into Eqs.(89-92), the analytical solution can be expressed:

$$\left(\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix}\right)\begin{bmatrix} U_{mn} \\ V_{mn} \\ W_{bmn} \\ W_{smn} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ F_{xy} \\ F_{xy} \end{bmatrix}$$

Where

$$\begin{aligned} s_{11} &= A_1\alpha^2 + C_1\beta^2 + h_1\left(\alpha^2\beta^2 - \alpha^4\right) \\ s_{12} &= A_1\upsilon\alpha\beta + C_1\alpha\beta + h_1\left(\alpha^3\beta - 0.5\alpha\beta^3\right) \\ s_{13} &= -B_1\left(\alpha^3 + \alpha\beta^2\right) + D_1\alpha\beta^2 \\ s_{14} &= 0 = s_{24} \\ s_{22} &= A_1\beta^2 + C_1\alpha^2 + \frac{h_1}{2}\left(2\alpha^4 + \alpha^2\beta^2\right) \\ s_{23} &= -B_1\beta^3 - B_1\upsilon\beta\alpha^2 + 2D_1\alpha^2\beta \\ s_{33} &= -U_3\left(\alpha^4 + \beta^4 + 2\alpha^2\beta^2\right) - \\ &\quad h_1\alpha^2\beta^2 - \frac{h_1}{2}\left(6\alpha^2\beta^2 + \alpha^4 + \beta^4\right) \\ s_{44} &= C_1\left(\alpha^2 + \beta^2\right) + \frac{h_1}{2}\left(4\alpha^2\beta^2 + \alpha^4 + \beta^4\right) \\ m_{11} &= I_0 = m_{22} = m_{43} = m_{44}, \\ m_{14} &= -\alpha I_1, m_{33} = I_0 + I_2\left(\alpha^2 + \beta^2\right), m_{23} = I_1\beta \end{aligned} \quad (94)$$

II DISCUSSION AND RESULTS

As mentioned in open literature, no research have been done applying modified couple stress theory to investigate vibration of Nano rectangular FG plate under moving load based on FSDT and SFSDT, so that the research evaluated with those were homogenous. This part divided into 2 sections; first studied the effects of power law index and length parameter scales on deflections and then on frequencies. Figure 3 shows the dimensionless deflection of Nano rectangular FG plate with different power law index and constant length scale parameters using FSDT. It is cleared by increasing of power law index, deflections also raised. Fig 6 indicate the plate's deflections in SFSDT as same as fig 3 conditions. The figure also has the identical trend like fig3. The reason of increasing deflection is, because metal purity increased than to ceramic, bending stiffness of elasticity module dropped which is led to more flexibility and eventually plate's deflection increased. Figure 4 indicates the dimensionless deflection of Nano rectangular FG plate by constant power law index and various length scale parameters in FSDT, fig 5 which is done based on SFSDT also has the fig 4 conditions. In figs 4&5 by enhancing of length scale parameters, plate's deflection reduced because with increasing of length scale parameters flexibility diminished so that the plate's deflection decreased. Either of next two branches figures, i.e. figs7&8(done regarding to FSDT) and figs9&10 (by considering SFSDT) express deflection of Nano rectangular FG plate during the time. It is noticeable that assumed load moves along x direction over the centerline of FG plate by a constant velocity $v_0(x=v_0t)$, also traveling time of the moving load defined as $t_f=\dfrac{a}{v_0}$, and k=5/6, so the T which **is** showed represented as: $T=\dfrac{t}{t_f}$ [21]. Figures of 8 and 9 shows deflection of the plate with constant power law index and different length scale parameters throughout the time, these images reveals that by increasing of length scale parameters deflections diminished, owing to dropping of bending stiffness of elasticity module. But the figs 7 &10 which power law indexes are varying, by rising of the factor, deflection raised, due to decreasing of flexibility. Figures 11 and 12 are according to FSDT and figs 13&14 represented SFSDT. Figures 11 to 14 probes the power law index and length scale parameters on frequencies, as can seen from the figures by increasing each of two factors, frequencies reduced. This might be because of small width of plate in Nano structure which is so significant and present such a trend.

III CONCLUSION

Comparison of vibration analysis of Nano rectangular FG plate applying to modified couple stress theory under moving load based on FSDT and SFSDT are developed. First, displacement fields are defined. Equations of motion derived from Hamilton's principle. Boundary conditions and analytical solution are stated for a simply supported Nano rectangular FG plate. FG properties vary through thickness. In order to investigation of small scale effects, modified couple stress is used which has one length scale parameter. Results show with power law index increasing, deflection escalated and rising of length scale parameter, deflection reduced, and these factors have same effects on frequencies. The research indicated that there is slightly difference between theories just at amplitudes.

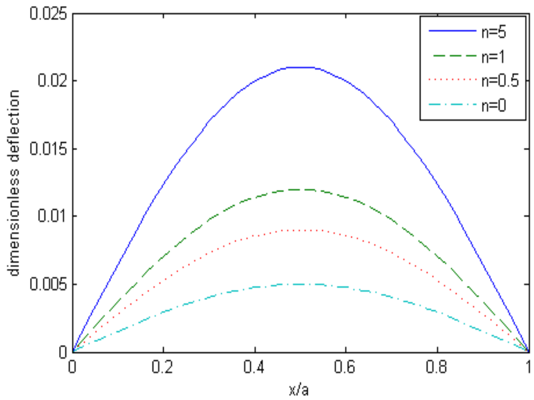

Figure 3. Dimensionless deflection of nano rectangular fg plate with different power law index and constant length scale parameter (h/l=1) at x_0=x and y_0=b/2

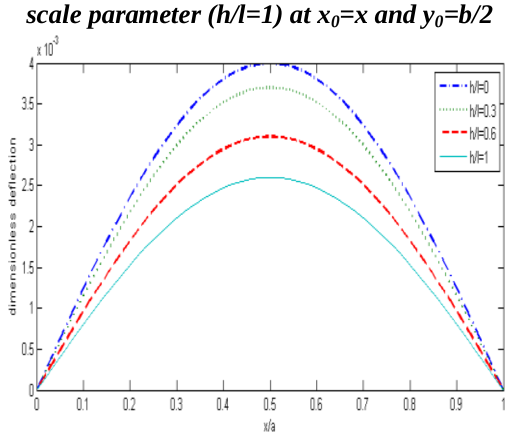

Figure 4. Dimensionless deflection of nano rectangular fg plate with constsnt power law index(n=1) and different length scale parameter at

x_0=x and y_0=b/2

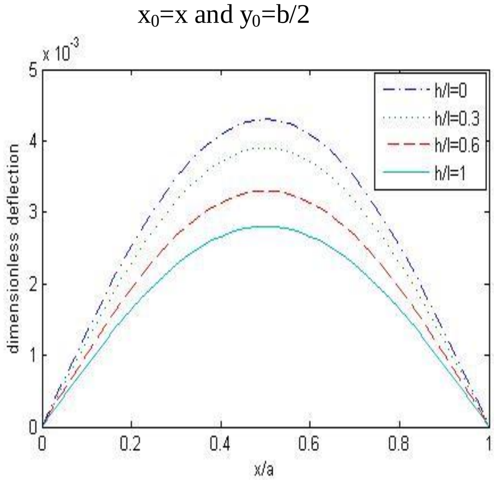

Figure 5. Dimensionless deflection of Nan rectangular FG plate with various length scale parameter and constant power law index (n=1) at

$$x_0 = x, y_0 = \frac{b}{2}$$

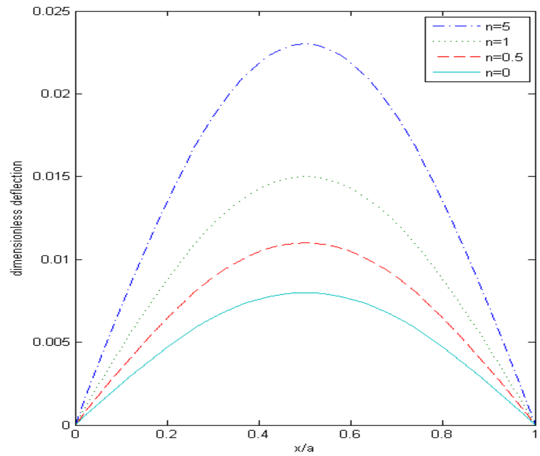

Figure 6. Dimensionless deflection of Nano rectangular FG plate with constant length scale parameter (h/l=1) and different power law index

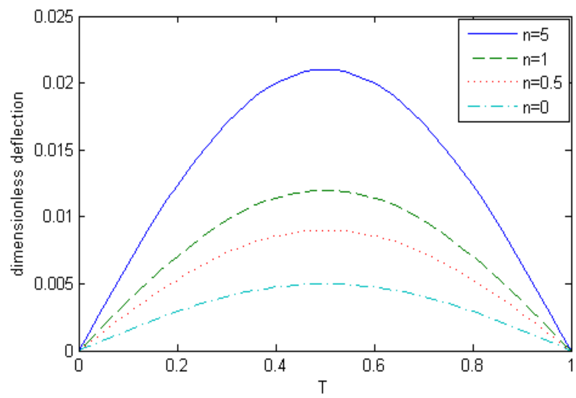

Figure 7. Dimensionless deflection with different power law index and constant length scale parameter(h/l=1) under moving load throughout of the time

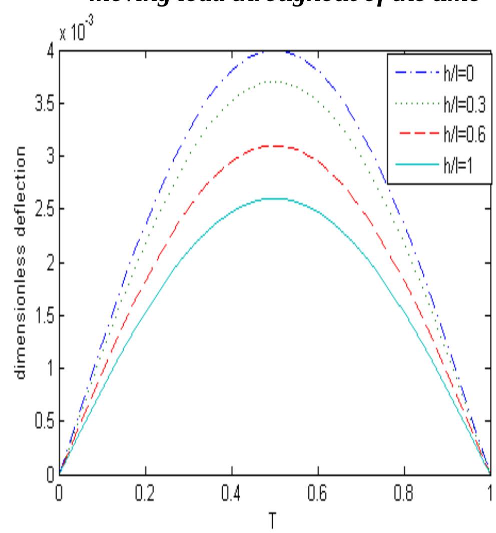

Figure 8. Dimensionless deflection with constant power law index(n=1) and different length scale parameters under moving load throughout of time

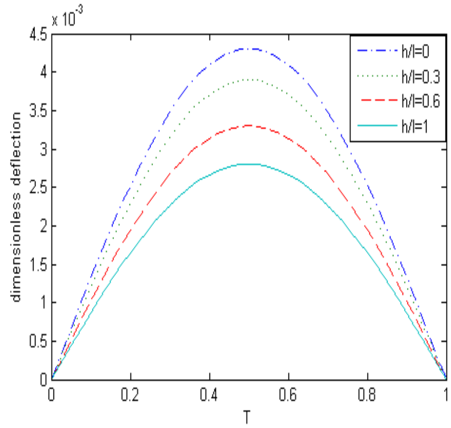

Figure 9. Dimensionless deflection with constant power law index (n=1) and different length scale parameter under moving load throughout of the time

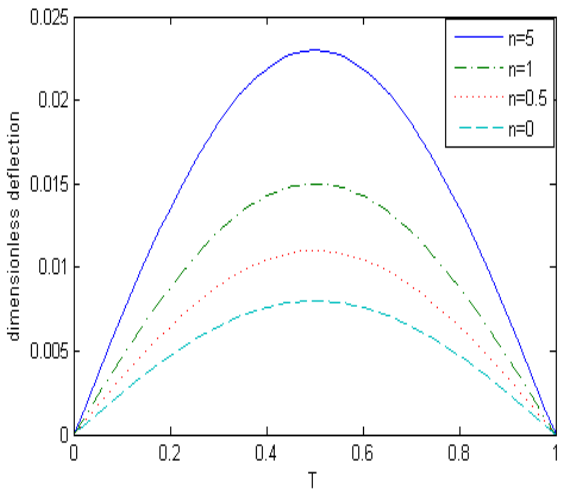

Figure 10. Dimensionless deflection with different power law index and constant length scale parameter (h/l=1) under moving load throughout of the time

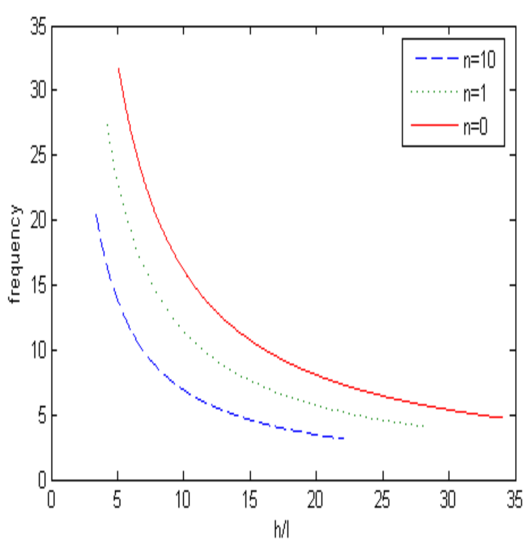

Figure 11. Effect of length scale parameter on Nano rectangular FG plate frequency

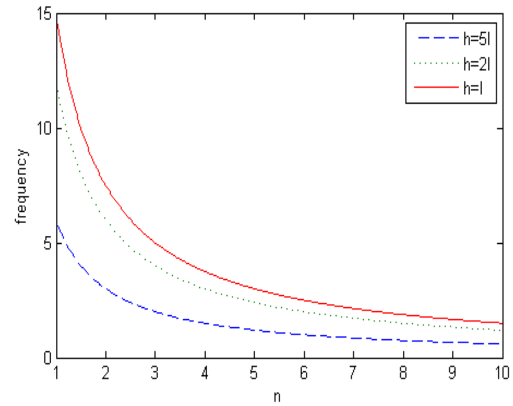

Figure 12. Effect of power law index on Nano rectangular FG plate frequency

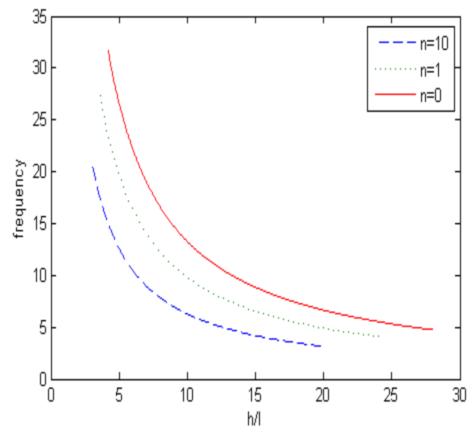

Figure 13. Effect of length scale parameter on Nano rectangular FG plate frequency

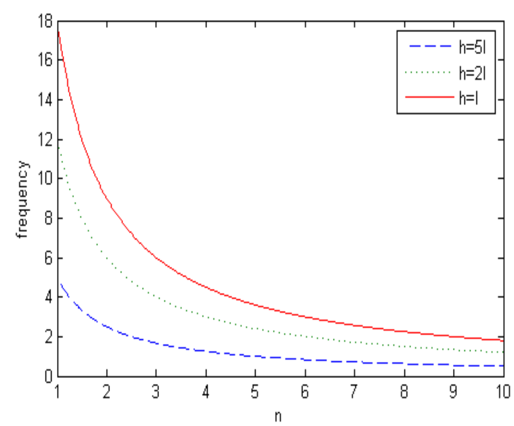

Figure 14. Effect of power law index on Nano rectangular FG plate frequency

REFERENCES

1. Zhang, B., et al., A non-classical Mindlin plate finite element based on a modified couple stress theory. European Journal of Mechanics-A/Solids, 2013. **42**: p. 63-80.

2. He, L., et al., A size-dependent four variable refined plate model for functionally graded microplates based on modified

couple stress theory. Composite Structures, 2015. **130**: p. 107-115.

3.Thai, H.-T. and S.-E. Kim, A size-dependent functionally graded Reddy plate model based on a modified couple stress theory. Composites Part B: Engineering, 2013. **45**(1): p. 1636-1645.

4.Vu, T.-V., et al., A simple FSDT-based meshfree method for analysis of functionally graded plates. Engineering Analysis with Boundary Elements, 2017. **79**: p. 1-12.

5.Jooybar, N., et al., Thermal effect on free vibration of functionally graded truncated conical shell panels. Thin-Walled Structures, 2016. **103**: p. 45-61.

6.Reddy, J., J. Romanoff, and J.A. Loya, Nonlinear finite element analysis of functionally graded circular plates with modified couple stress theory. European Journal of Mechanics-A/Solids, 2016. **56**: p. 92-104.

7.Mirsalehi, M., M. Azhari, and H. Amoushahi, Stability of thin FGM microplate subjected to mechanical and thermal loading based on the modified couple stress theory and spline finite strip method. Aerospace Science and Technology, 2015. **47**: p. 356-366.

8.Lei, J., et al., A size-dependent FG micro-plate model incorporating higher-order shear and normal deformation effects based on a modified couple stress theory. International Journal of Mechanical Sciences, 2015. **104**: p. 8-23.

9.Thai, S., et al., A simple shear deformation theory for nonlocal beams. Composite Structures, 2017.

10.Gupta, A. and M. Talha, Nonlinear flexural and vibration response of geometrically imperfect gradient plates using hyperbolic higher-order shear and normal deformation theory. Composites Part B: Engineering, 2017. **123**: p. 241-261.

11.Senjanović, I., N. Vladimir, and M. Tomić, On new first-order shear deformation plate theories. Mechanics research communications, 2016. **73**: p. 31-38.

12.Senjanović, I., et al., New first order shear deformation beam theory with in-plane shear influence. Engineering structures, 2016. **110**: p. 169-183.

13.Yin, S., et al., Isogeometric locking-free plate element: a simple first order shear deformation theory for functionally graded plates. Composite Structures, 2014. **118**: p. 121-138.

14.Rohwer, K., Remarks on ‘‘A simple first-order shear deformation theory for laminated composite plates’’. Composite Structures, 2013.

15.Belabed, Z., et al., An efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates. Composites Part B: Engineering, 2014. **60**: p. 274-283.

16.Thai, H.-T. and D.-H. Choi, A simple first-order shear deformation theory for laminated composite plates. Composite Structures, 2013. **106**: p. 754-763.

17.Thai, H.-T. and S.-E. Kim, A simple higher-order shear deformation theory for bending and free vibration analysis of functionally graded plates. Composite Structures, 2013. **96**: p. 165-173.

18.Thai, H.-T. and D.-H. Choi, A simple first-order shear deformation theory for the bending and free vibration analysis of functionally graded plates. Composite Structures, 2013. **101**: p. 332-340.

19.Thai, H.-T., et al., Analysis of functionally graded sandwich plates using a new first-order shear deformation theory. European Journal of Mechanics-A/Solids, 2014. **45**: p. 211-225.

20.Thai, H.-T. and T.P. Vo, A size-dependent functionally graded sinusoidal plate model based on a modified couple stress theory. Composite Structures, 2013. **96**: p. 376-383.

21.Malekzadeh, P. and S. Monajjemzadeh, Dynamic response of functionally graded plates in thermal environment under moving load. Composites Part B: Engineering, 2013. **45**(1): p. 1521-1533.

22.Thai, H.-T., et al., Size-dependent behavior of functionally graded sandwich microbeams based on the modified couple stress theory. Composite Structures, 2015. **123**: p. 337-349.

23.Hao, Y., W. Zhang, and J. Yang, Nonlinear dynamics of a FGM plate with two clamped opposite edges and two free edges. Acta Mechanica Solida Sinica, 2014. **27**(4): p. 394-406.

24.Salehipour, H., H. Nahvi, and A. Shahidi, Exact closed-form free vibration analysis for functionally graded micro/nano plates based on modified couple stress and three-dimensional elasticity theories. Composite Structures, 2015. **124**: p. 283-291.

25.Kim, J. and J. Reddy, Analytical solutions for bending, vibration, and buckling of FGM plates using a couple stress-based third-order theory. Composite Structures, 2013. **103**: p. 86-98.

26.Reddy, J.N., Theory and analysis of elastic plates and shells. 2006: CRC press.

27.Lou, J. and L. He, Closed-form solutions for nonlinear bending and free vibration of functionally graded microplates based on the modified couple stress theory. Composite Structures, 2015. **131**: p. 810-820.