OPEN ACCESS INTERNATIONAL JOURNAL OF SCIENCE & ENGINEERING

Numerical Assessment of Damage Localisation and Severity in Reinforced Concrete Beams Using Modal Analysis

Harshit Verma[1], Mr. Sumit Pahwa

[1]*M. Tech Scholar, Department of Civil Engineering, Alpine Institute of Technology, Ujjain. (Harshit.k0710@gmail.com)*

Abstract:Structural damage in reinforced concrete beams leads to stiffness degradation, which alters the dynamic characteristics of the structure. Early identification of such damage is essential for ensuring structural safety and serviceability. This paper presents a numerical investigation of modal-based damage identification in reinforced concrete beams using vibration characteristics obtained from finite element analysis. A simply supported reinforced concrete beam is modelled using the finite element method, and damage is simulated by introducing localized stiffness reduction in a selected beam element.

Modal analysis is performed for healthy and damaged beam configurations to extract natural frequencies and mode shapes. Damage identification is carried out using three vibration-based indicators: natural frequency variation for damage detection, mode shape curvature for damage localisation, and modal strain energy for damage severity assessment. Multiple damage scenarios corresponding to 10%, 20%, and 40% stiffness reduction are considered to evaluate the sensitivity of the adopted methods.

The numerical results indicate a consistent reduction in natural frequencies with increasing damage severity, confirming their effectiveness in detecting the presence of damage. However, frequency-based indicators alone are insufficient for precise damage localisation. The mode shape curvature method successfully identifies the location of damage along the beam length for all damage cases, while the modal strain energy method provides a reliable quantitative measure of damage severity. A comparative assessment demonstrates that the combined application of mode shape curvature and modal strain energy methods offers a robust and comprehensive framework for vibration-based damage identification in reinforced concrete beams.

Keywords: Structural health monitoring; Reinforced concrete beam; Modal analysis; Damage identification; Mode shape curvature; Modal strain energy; Finite element method

---

I.INTRODUCTION

Reinforced concrete (RC) beams constitute critical load-carrying elements in civil infrastructure such as buildings, bridges, and industrial structures. Over time, these structural components are subjected to deterioration due to factors including material ageing, fatigue loading, environmental exposure, corrosion of reinforcement, and accidental overloading. Such degradation results in stiffness reduction and progressive damage, which may compromise structural safety and serviceability if not identified at an early stage. Consequently, reliable damage identification techniques are essential for effective structural health monitoring (SHM) of reinforced concrete structures.

Conventional damage assessment methods, including visual inspection and localized non-destructive testing, are often labor-intensive, subjective, and limited to accessible regions of the structure. These approaches may fail to detect internal or early-stage damage, particularly in large-scale civil structures. To overcome these limitations, vibration-based damage identification techniques have gained significant attention due to their global sensitivity to structural changes and their ability to assess the overall condition of a structure using measured dynamic response data.

Vibration-based methods rely on the principle that structural damage alters physical properties such as stiffness, mass, and damping, which in turn affects modal parameters including natural frequencies, mode shapes, and modal strain energy distribution. Among these parameters, changes in natural frequencies are commonly used as preliminary indicators of damage because they are relatively easy to extract and are less sensitive to measurement noise. However, frequency-based methods alone are generally insufficient for accurate damage localisation, as different damage scenarios may produce similar frequency shifts.

To address this limitation, mode shape-based techniques have been extensively investigated for damage localisation. In particular, mode shape curvature has been shown to be highly sensitive to local stiffness variations in beam-like structures, enabling effective identification of damage location. Similarly, modal strain energy-based methods exploit changes in the energy distribution of vibration modes to quantify damage severity. These approaches have demonstrated improved sensitivity to localized damage compared to frequency-based indicators and are well suited for numerical and experimental SHM studies.

Despite extensive research on individual vibration-based indicators, practical damage identification remains challenging due to the complementary nature of detection, localisation, and severity assessment tasks. A single modal parameter is often insufficient to address all aspects of damage identification reliably. Therefore, combining multiple modal-based indicators within a unified framework can enhance robustness and improve damage assessment-accuracy.

II.NUMERICAL MODELLING AND METHODOLOGY

2.1 Finite Element Modelling of Reinforced Concrete Beam

A simply supported reinforced concrete beam is considered in this study to investigate vibration-based damage identification. The beam is numerically modelled using the finite element method under the assumption of linear elastic behaviour, which is appropriate for modal analysis and small vibration amplitudes. The reinforced concrete section is idealised as an equivalent homogeneous beam, and the effects of reinforcement are incorporated through the effective flexural rigidity of the section.

The beam is discretised into a number of one-dimensional beam elements based on Euler–Bernoulli beam theory, which assumes plane sections remain plane and neglects shear deformation effects. Each node possesses two degrees of freedom corresponding to transverse displacement and rotation. The global mass and stiffness matrices of the beam are assembled from the element-level matrices using standard finite element procedures. Simply supported boundary conditions are applied by restraining vertical displacement at both supports while allowing rotational freedom.

2.2 Modal Analysis of the Healthy Beam

Modal analysis is first performed on the healthy beam model to establish baseline dynamic characteristics. The undamped free vibration equation of motion is expressed in matrix form, and the corresponding eigenvalue problem is solved to obtain natural frequencies and mode shapes. The first few lower vibration modes are extracted, as they are known to be more sensitive to structural damage and are commonly used in vibration-based damage identification studies.

The obtained natural frequencies and mode shapes of the healthy beam serve as reference data for subsequent damage assessment. These baseline modal parameters are also used to validate the numerical model by ensuring physically consistent mode shapes and reasonable frequency values.

2.3 Damage Modelling Using Stiffness Reduction

Structural damage is simulated numerically by introducing localized stiffness reduction in a selected finite element of the beam. This approach represents the effect of cracking, material degradation, or loss of load-carrying capacity commonly observed in reinforced concrete structures. Damage is modelled by reducing the flexural stiffness of the damaged element while keeping the mass matrix unchanged, as mass variation due to damage is generally negligible compared to stiffness loss.

To investigate the influence of damage severity, multiple damage scenarios are considered by applying stiffness reduction levels of 10%, 20%, and 40% at the same damage location. These damage cases allow evaluation of the sensitivity of modal parameters and damage indices to varying degrees of structural deterioration.

2.4 Frequency-Based Damage Detection

Natural frequency variation is used as a preliminary damage detection indicator. Modal analysis is performed for each damaged configuration, and the natural frequencies are compared with those of the healthy beam. A reduction in natural frequency is expected due to stiffness degradation caused by damage. Although frequency changes provide useful information regarding the presence of damage, they do not offer sufficient spatial resolution to accurately locate the damaged region. Therefore, frequency-based detection is employed only as an initial diagnostic tool in this study.

2.5 Mode Shape Curvature-Based Damage Localisation

To identify the location of damage, the mode shape curvature (MSC) method is employed. Mode shape curvature is computed by taking the second spatial derivative of the mode shapes, which enhances sensitivity to local stiffness changes. In this study, numerical differentiation using the central finite difference scheme is adopted to calculate curvature values along the beam length.

The curvature of the damaged beam is compared with that of the healthy beam, and the difference between the two is evaluated. A pronounced peak in the curvature difference indicates the presence and location of damage. The MSC method is applied to the first few vibration modes to ensure reliable localisation results, as higher modes may be more sensitive to noise and numerical errors.

2.6 Modal Strain Energy-Based Damage Severity Assessment

Modal strain energy (MSE) is used to quantify damage severity. Modal strain energy represents the distribution of deformation energy associated with a vibration mode and is directly related to structural stiffness. Damage causes redistribution of strain energy, particularly in the vicinity of the damaged element.

In this study, the modal strain energy of each beam element is computed for both healthy and damaged conditions. A damage index based on the relative change in modal strain energy is evaluated for each element. Higher damage index values indicate greater stiffness degradation. By analysing the peak values of the damage index for different stiffness reduction levels, the effectiveness of the MSE method in assessing damage severity is examined.

2.7 Overall Damage Identification Framework

The overall damage identification methodology adopted in this study follows a sequential approach. Natural frequency variation is first used to detect the presence of damage. Mode shape curvature is then employed to accurately locate the damaged region along the beam. Finally, modal strain energy analysis is used to assess the severity of damage quantitatively. This combined approach leverages the complementary strengths of individual modal parameters and provides a comprehensive framework for vibration-based damage identification in reinforced concrete beams.

2.8 Mathematical Formulation of Modal-Based Damage Identification

A. Governing Equation of Motion

For an undamped linear structural system, the free vibration equation of motion is given by

$M \ddot{x}(t) + K x(t) = 0$ (1)

Where M is the global mass matrix, K is the global stiffness matrix, and x(t) is the displacement vector.

Assuming a harmonic solution of the form

$x(t) = \varphi \sin(\omega t)$ (2)

substitution into Eq. (1) leads to the standard eigenvalue problem

$(K - \omega^2 M) \varphi = 0$ (3)

Where ω is the natural circular frequency, and φ is the corresponding mode shape vector.

B. Natural Frequency

The natural frequency in Hertz is obtained from the circular frequency as

$f = \omega / (2\pi)$ (4)

These natural frequencies and mode shapes form the basis for vibration-based damage identification.

C. Euler–Bernoulli Beam Governing Equation

The transverse vibration of an Euler–Bernoulli beam is governed by

$EI\, \partial^4 w(x,t)/\partial x^4 + \rho A\, \partial^2 w(x,t)/\partial t^2 = 0$ (5)

Where E is Young's modulus, I is the second moment of area, ρ is the mass density, A is the cross-sectional area, and w(x,t) is the transverse displacement.

This formulation is used to derive beam element stiffness and mass matrices in the finite element model.

D. Damage Modelling Using Stiffness Reduction

Structural damage is simulated by reducing the flexural stiffness of the damaged element as

$K_d = (1 - \alpha) K_h$ (6)

Where K_h is the stiffness matrix of the healthy element, K_d is the stiffness matrix of the damaged element, and α represents the damage severity factor ($0 < \alpha < 1$).

In this study, α = 0.1, 0.2, and 0.4 correspond to 10%, 20%, and 40% stiffness reduction, respectively.

E. Mode Shape Curvature (MSC)

Mode shape curvature is defined as the second spatial derivative of the mode shape:

$\kappa_i(x) = d^2\varphi_i(x) / dx^2$ (7)

In numerical implementation, curvature at node j is approximated using the central finite difference scheme:

$\kappa_i(j) \approx [\ \varphi_i(j+1) - 2\varphi\_i(j) + \varphi_i(j-1)\ ] / \Delta x^2$ (8)

where Δx is the element length.

The damage index based on curvature difference is computed as

$DI_{MSC}(j) = |\ \kappa_i\hat{}d(j) - \kappa_i\hat{}h(j)\ |$ (9)

where superscripts h and d denote healthy and damaged states, respectively.

F. Modal Strain Energy (MSE)

The modal strain energy of the i-th mode for element e is expressed as

$MSE_i\hat{}e = (1/2) \int_e EI\ [\ d^2\varphi_i(x)/dx^2\ ]^2\ dx$ (10)

The modal strain energy-based damage index is defined as

$DI_{MSE}\hat{}e = |\ MSE_i\hat{}d(e) - MSE_i\hat{}h(e)\ | / MSE_i\hat{}h(e)$ (11)

Higher values of DI_{MSE} indicate greater stiffness degradation and damage severity.

III.RESULTS AND DISCUSSION

3.1 Modal Characteristics of the Healthy Reinforced Concrete Beam

Modal analysis is initially performed on the healthy reinforced concrete beam to establish baseline dynamic characteristics. The extracted natural frequencies and corresponding mode shapes exhibit physically consistent flexural behaviour, confirming the correctness of the finite element model, material properties, and boundary condition implementation. The lower vibration modes show smooth spatial deformation patterns and are dominated by global bending, making them suitable for vibration-based damage identification.

These baseline modal parameters serve as reference data for all subsequent damage scenarios. In particular, the first few modes are retained for further analysis, as higher modes are generally more sensitive to numerical noise and less reliable for damage interpretation.

3.2 Effect of Damage Severity on Natural Frequencies

The variation of natural frequencies with damage severity is analysed by comparing the modal properties of healthy and damaged beam configurations. Damage is simulated through localized stiffness reduction, and the corresponding changes in natural frequencies are evaluated.

The results indicate a monotonic reduction in natural frequencies with increasing damage severity. The first natural frequency shows the highest sensitivity to stiffness degradation, while higher modes exhibit comparatively smaller variations.

This behaviour confirms the direct relationship between structural stiffness and dynamic response. Although frequency reduction clearly indicates the presence of damage, it does not provide sufficient information regarding the location of damage.

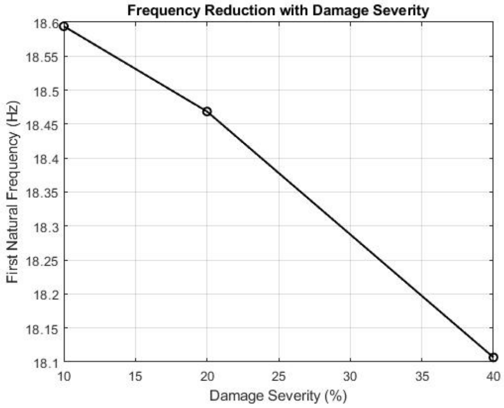

Figure 1 presents the variation of the first natural frequency with different damage severity levels .

3.3 Damage Localisation Using Mode Shape Curvature

To identify the spatial location of damage, the mode shape curvature (MSC) method is applied. Mode shape curvature is computed using numerical differentiation of mode shapes obtained from modal analysis. The curvature difference between healthy and damaged beam configurations is evaluated along the beam length.

The results demonstrate that the curvature difference exhibits a distinct peak at the damaged element location for all considered damage severities. As damage severity increases, the magnitude of the curvature peak increases, while its location remains unchanged. This confirms that the MSC method is highly effective for damage localisation but primarily provides qualitative information regarding damage presence rather than severity.

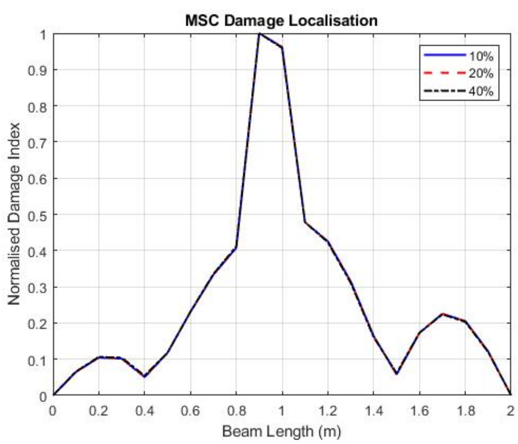

Figure 2 shows the mode shape curvature-based damage localisation plot

3.4 Damage Severity Assessment Using Modal Strain Energy

Modal strain energy (MSE) is employed to quantify damage severity. The modal strain energy distribution is computed for each beam element under healthy and damaged conditions. Damage indices are evaluated based on the relative change in modal strain energy.

The results reveal a clear redistribution of strain energy towards the damaged element as stiffness reduction increases. The peak modal strain energy damage index increases significantly with damage severity, enabling effective differentiation between low, moderate, and severe damage cases. Unlike frequency-based indicators, the MSE method provides a quantitative measure of damage severity.

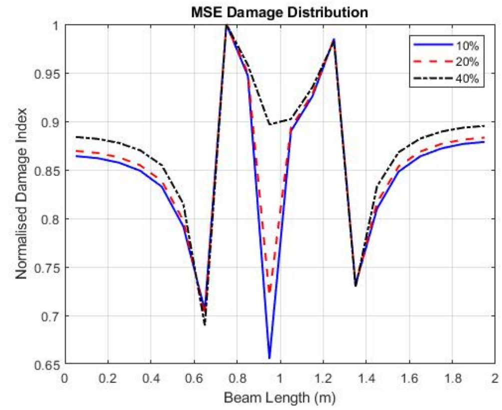

Figure 3 illustrates the modal strain energy-based damage distribution along the beam for different damage severity levels

3.5 Comparative Assessment of Damage Indicators

A comparative analysis of the different damage indicators highlights their complementary characteristics. Natural frequency variation is effective for detecting the presence of damage but lacks spatial resolution. The mode shape curvature method accurately localises damage but does not directly quantify severity. In contrast, the modal strain energy method provides reliable severity estimation but benefits from prior localisation information.

To further examine the sensitivity of the adopted methods, peak damage index values obtained from MSC and MSE analyses are compared across different damage severity levels. The results show a consistent increase in peak damage index values with increasing stiffness reduction, with the MSE method exhibiting higher sensitivity to damage severity.

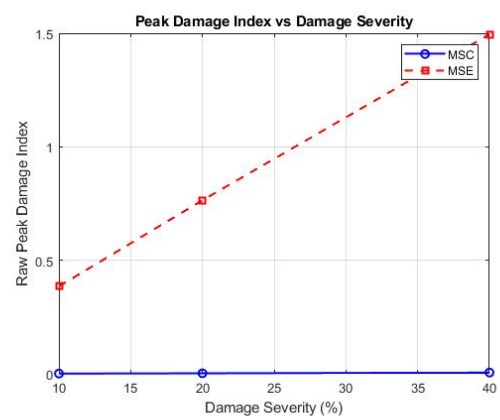

Figure 4 presents the comparison of peak damage index values with damage severity

IV.DISCUSSION

The numerical results confirm that vibration-based damage identification techniques are effective for reinforced concrete beams under linear elastic conditions. The combined use of natural frequency variation, mode shape curvature, and modal strain energy enables reliable detection, localisation, and severity assessment of damage. The use of MATLAB-generated plots ensures consistency between numerical computation and result interpretation.

The findings demonstrate that relying on a single modal indicator may lead to incomplete damage assessment, whereas an integrated modal-based framework significantly enhances reliability and robustness. These observations are consistent with trends reported in the literature and highlight the practical potential of the proposed methodology for structural health monitoring applications.

V.CONCLUSIONS

This paper presented a numerical investigation of modal-based damage identification in reinforced concrete beams using vibration characteristics obtained from finite element analysis. Structural damage was simulated through localized stiffness reduction, and the effectiveness of natural frequency variation, mode shape curvature, and modal strain energy methods was evaluated for damage detection, localisation, and severity assessment.

The numerical results demonstrated that natural frequency reduction is a reliable indicator for detecting the presence of damage, with the first vibration mode exhibiting the highest sensitivity to stiffness degradation. However, frequency-based indicators alone were found to be insufficient for accurate damage localisation due to their global nature and limited spatial resolution.

The mode shape curvature method showed strong capability in accurately localising the damaged region along the beam length for all considered damage severities. Distinct curvature peaks were consistently observed at the damaged element location, confirming the robustness of the MSC approach for spatial damage identification. Nevertheless, the MSC method provides primarily qualitative information and does not directly quantify damage severity.

VI.REFERENCES

[1] C. R. Farrar and K. Worden, “An introduction to structural health monitoring,” *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 365, no. 1851, pp. 303–315, Jan. 2007, doi: 10.1098/rsta.2006.1928.

[2] H. Sohn, C. R. Farrar, F. M. Hemez, D. D. Shunk, D. W. Stinemates, B. R. Nadler, and J. J. Czarnecki, *A Review of Structural Health Monitoring Literature: 1996–2001*, Los Alamos National Laboratory Report LA-13976-MS, 2004.

[3] A. K. Pandey, M. Biswas, and M. M. Samman, “Damage detection from changes in curvature mode shapes,” *Journal of Sound and Vibration*, vol. 145, no. 2, pp. 321–332, Apr. 1991, doi: 10.1016/0022-460X(91)90595-B.

[4] S. W. Doebling, C. R. Farrar, M. B. Prime, and D. W. Shevitz, “Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: A literature review,” Los Alamos National Laboratory Report LA-13070-MS, 1996.

[5] M. Morassi and S. Tonon, “Dynamic methods for damage detection in structures: A survey,” *Applied Mechanics Reviews*, vol. 53, no. 3, pp. 81–100, Mar. 2000, doi: 10.1115/1.3097349.

[6] O. S. Salawu, “Detection of structural damage through changes in frequency: A review,” *Engineering Structures*, vol. 19, no. 9, pp. 718–723, Sep. 1997, doi: 10.1016/S0141-0296(96)00149-6.

[7] J. M. W. Brownjohn, “Structural health monitoring of civil infrastructure,” *Philosophical Transactions of the Royal Society A*, vol. 365, no. 1851, pp. 589–622, Jan. 2007, doi: 10.1098/rsta.2006.1925.

[8] A. Rytter, “Vibration based inspection of civil engineering structures,” Ph.D. dissertation, Dept. of Building Technology and Structural Engineering, Aalborg Univ., Denmark, 1993.

[9] S. W. Doebling, C. R. Farrar, and M. B. Prime, “A summary review of vibration-based damage identification methods,” *The Shock and Vibration Digest*, vol. 30, no. 2, pp. 91–105, Mar. 1998, doi: 10.1177/058310249803000201.

[10] K. Worden and J. Dulieu-Barton, “An overview of intelligent fault detection in systems and structures,” *Structural Health Monitoring*, vol. 3, no. 1, pp. 85–98, Mar. 2004, doi: 10.1177/1475921704041866.

[11] O. S. Salawu, “Detection of structural damage through changes in frequency: A review,” *Engineering Structures*, vol. 19, no. 9, pp. 718–723, Sep. 1997, doi: 10.1016/S0141-0296(96)00149-6.

[12] A. Rytter, “Vibration based inspection of civil engineering structures,” Ph.D. dissertation, Aalborg Univ., Denmark, 1993.

[13] M. Abdel Wahab and G. De Roeck, “Damage detection in bridges using modal curvatures: Application to a real bridge,” *Journal of Sound and Vibration*, vol. 226, no. 2, pp. 217–235, Sep. 1999, doi: 10.1006/jsvi.1999.2345.

[14] J. Maeck and G. De Roeck, “Damage assessment using vibration analysis on civil engineering structures,” *Engineering Structures*, vol. 25, no. 7, pp. 867–879, Jun. 2003, doi: 10.1016/S0141-0296(03)00033-5.

[15] A. K. Pandey, M. Biswas, and M. M. Samman, “Damage detection from changes in curvature mode shapes,” *Journal of Sound and Vibration*, vol. 145, no. 2, pp. 321–332, Apr. 1991, doi: 10.1016/0022-460X(91)90595-B.

[16] Z. Y. Shi, S. S. Law, and L. M. Zhang, “Damage localization by directly using mode shape curvatures,” *Journal of Engineering Mechanics*, vol. 126, no. 7, pp. 656–664, Jul. 2000, doi: 10.1061/(ASCE)0733-9399(2000)126:7(656).

[17] S. Stubbs, J. T. Kim, and C. R. Farrar, “Field verification of a non-destructive damage localization and severity estimation algorithm,” in *Proc. 13th Int. Modal Analysis Conf.*, Nashville, TN, USA, 1995, pp. 210–218.

[18] J. T. Kim and S. Stubbs, “Improved damage identification method based on modal information,” *Journal of Sound and Vibration*, vol. 252, no. 2, pp. 223–238, Apr. 2002, doi: 10.1006/jsvi.2001.3749.

[19] C. R. Farrar and K. Worden, *Structural Health Monitoring: A Machine Learning Perspective*, John Wiley & Sons, 2013.

[20] H. Sohn et al., "A review of structural health monitoring literature: 2001–2006," Los Alamos National Laboratory Report LA-14376-MS, 2007.

[21] R. W. Clough and J. Penzien, *Dynamics of Structures*, 3rd ed., Berkeley, CA, USA: Computers & Structures Inc., 2003.

[22] A. K. Chopra, *Dynamics of Structures: Theory and Applications to Earthquake Engineering*, 4th ed., Upper Saddle River, NJ, USA: Prentice Hall, 2012.

[23] S. S. Rao, *Mechanical Vibrations*, 6th ed., Upper Saddle River, NJ, USA: Pearson Education, 2017.

[24] L. Meirovitch, *Elements of Vibration Analysis*, 2nd ed., New York, NY, USA: McGraw-Hill, 1986.

[25] S. Timoshenko, D. H. Young, and W. Weaver, *Vibration Problems in Engineering*, 4th ed., New York, NY, USA: John Wiley & Sons, 1974.

[26] W. Weaver Jr., S. P. Timoshenko, and D. H. Young, *Vibration Problems in Engineering*, 5th ed., New York, NY, USA: John Wiley & Sons, 1990.

[27] O. C. Zienkiewicz, R. L. Taylor, and J. Z. Zhu, *The Finite Element Method: Its Basis and Fundamentals*, 7th ed., Oxford, U.K.: Butterworth-Heinemann, 2013.

[28] J. N. Reddy, *An Introduction to the Finite Element Method*, 3rd ed., New York, NY, USA: McGraw-Hill, 2006.

[29] K. J. Bathe, *Finite Element Procedures*, Upper Saddle River, NJ, USA: Prentice Hall, 1996.

[30] D. J. Ewins, *Modal Testing: Theory, Practice and Application*, 2nd ed., Baldock, U.K.: Research Studies Press, 2000.

[31] M. Géradin and D. Rixen, *Mechanical Vibrations: Theory and Application to Structural Dynamics*, 3rd ed., Chichester, U.K.: John Wiley & Sons, 2015.

[32] S. W. Doebling, C. R. Farrar, and M. B. Prime, "Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: A literature review," Los Alamos National Laboratory Report LA-13070-MS, 1996.

[33] J. T. Kim and S. Stubbs, "Improved damage identification method based on modal information," *Journal of Sound and Vibration*, vol. 252, no. 2, pp. 223–238, Apr. 2002, doi: 10.1006/jsvi.2001.3749.

[34] Z. Y. Shi, S. S. Law, and L. M. Zhang, "Damage localization by directly using mode shape curvatures," *Journal of Engineering Mechanics*, vol. 126, no. 7, pp. 656–664, Jul. 2000, doi: 10.1061/(ASCE)0733-9399(2000)126:7(656).

[35] A. K. Pandey, M. Biswas, and M. M. Samman, "Damage detection from changes in curvature mode shapes," *Journal of Sound and Vibration*, vol. 145, no. 2, pp. 321–332, Apr. 1991, doi: 10.1016/0022-460X(91)90595-B.